Szétválasztható:

$$y'(x) = e^{y(x) - x}$$

$$y'(x) = e^{y(x)} \cdot e^{-x}$$

$$y' = e^{y} \cdot e^{-x} \quad /:e^{y}$$

$$e^{-x} - 1 > 0$$

$$e^{-x} > 1$$

$$-x > 0$$

$$x < 0$$

$$e^{-y} \frac{dy}{dx} = e^{-x} \quad /\cdot dx$$

$$e^{-y} dy = e^{-x} dx$$

$$\int e^{-y} dy = \int e^{-x} dx$$

$$-e^{-y} = -e^{-x} + c \quad /:(-1)$$

$$-y = \ln\left(e^{-x} + c\right)$$

Hiányos másodfokú:

$$y''(x) = y'(x)\, e^{y(x)}$$

$$y'(0) = 1 \qquad y(0) = 0$$

$$y'(x) = p(y(x)) \qquad x = 0 \qquad 1 = p(0)$$

$$p'(y(x))\, p(y(x)) = p(y(x))\, e^{y(x)}$$

$$p'(x)\, p(x) = p(x) \cdot e^{x}$$

$$p'(x) = e^{x}$$

$$p(x) = e^{x} + C \Rightarrow C = 0$$

$$p(x) = e^{x}$$

$$y'(x) = e^{y(x)}$$

$$\frac{dy}{dx} = e^{y} \qquad /:e^{y};\ \cdot dx$$

$$\int e^{-y} dy = \int dx$$

$$-e^{-y} = x + C$$

$$-e^{-y} = x + k$$

$$\mapsto k = -1$$

$$e^{-y} = 1 - x$$

$$y(x) = -\ln(1 - x)$$

(1) szétválasztható:

$$y'(x) = y(x)$$

$$\frac{dy}{dx} = y$$

$$\int \frac{1}{y}\,dy = \int dx$$

$$\ln y = x + c$$

$$y = e^{x+c}$$

$$\underline{\underline{y = e^x \cdot C}}$$

(2) szétválasztható:

$$y'(x) = x y(x)$$

$$y' = xy$$

$$\frac{dy}{dx} = xy$$

$$\int \frac{1}{y}\,dy = \int x\,dx$$

$$\ln y = \frac{x^2}{2} + c$$

$$y = e^{\frac{x^2}{2} + C}$$

$$\underline{\underline{y = e^{\frac{x^2}{2}} \cdot c}}$$

(3) szétválasztható:

$$y'(x) = 4x\sqrt{y(x)} \qquad y(1) = 1$$

$$y' = 4x\, y^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 4x y^{\frac{1}{2}} \qquad /: y^{\frac{1}{2}}$$

$$\int y^{-\frac{1}{2}}\,dy = \int 4x\,dx$$

$$\frac{y^{\frac{1}{2}}}{\frac{1}{2}} = 4\frac{x^2}{2} + c$$

$$2y^{\frac{1}{2}} = 2x^2 + c \quad /:2$$

$$y^{\frac{1}{2}} = x^2 + \frac{c}{2}$$

$$y^{\frac{1}{2}} = x^2 + c \qquad /()^2$$

$$\underline{\underline{y = (x^2 + c)^2}}$$

(4) szétválasztható:

$$y'(x) = x e^{y(x)}$$

$$y' = x e^y$$

$$\frac{dy}{dx} = x e^y$$

$$\int e^{-y}\,dy = \int x\,dx$$

$$-e^{-y} = \frac{x^2}{2} + c$$

$$e^{-y} = \frac{-x^2}{2} + c$$

$$-y = \ln\left(-\frac{x^2}{2} + c\right) \qquad -\frac{x^2}{2} + c \neq 0 \qquad \frac{x^2}{2} \neq c$$

$$y = -\ln\left(-\frac{x^2}{2} + c\right)$$

$$y = \ln\left(-\frac{x^2}{2} + c\right)^{-1}$$

$$y = \ln \frac{1}{-\frac{x^2}{2} + c}$$

$$y = \ln \frac{2}{-x^2 + c}$$

$$\underline{\underline{y = \ln \frac{2}{c - x^2}}}$$

6 **változóiban homogén:**

$$y'(x) = \frac{y(x)}{y(x)+x} \qquad y+x \neq 0 \qquad y \neq -x$$

$$y' = \frac{y}{y+x}$$

$$y' = \frac{\frac{y}{x}}{\frac{y}{x}+1}$$

$$u = \frac{y}{x} \qquad y = ux \qquad y' = u'x + u$$

$$u'x + u = \frac{u}{u+1}$$

$$u'x = \frac{u}{u+1} - u$$

$$u'x = \frac{u}{u+1} - \frac{u^2+u}{u+1}$$

$$u'x = \frac{\cancel{u} - u^2 \cancel{- u}}{u+1} = \frac{-u^2}{u+1}$$

$$u'x = \frac{-u^2}{u+1} \qquad \Big/ : x \quad : \frac{u^2}{u+1}$$

$$\frac{u+1}{u^2}\, u' = -\frac{1}{x}$$

$$\int \frac{u+1}{u^2}\, du = -\int \frac{1}{x}\, dx \qquad \int u^{-2} = \frac{u^{-2+1}}{-2+1} = \frac{u^{-1}}{-1}$$

$$\int \left(\frac{1}{u} + \frac{1}{u^2}\right) du = -\int \frac{1}{x}\, dx$$

$$\int \frac{1}{u}\, du + \int u^{-2}\, du = -\int \frac{1}{x}\, dx$$

$$\ln|u| + \frac{u^{-1}}{-1} = -\ln|x| + C$$

$$\ln|u| - \frac{1}{u} = -\ln|x| + C$$

$$\ln\left|\frac{y}{x}\right| - \frac{x}{y} = -\ln|x| + C$$

$$\ln|y| - \ln|x| - \frac{x}{y} = -\ln|x| + C$$

$$\ln|y| = \frac{x}{y} + C$$

$$e^{\ln|y|} = e^{\frac{x}{y} + C}$$

$$|y| = e^{\frac{x}{y}} \cdot e^{C}$$

$$y = \pm\, e^{\frac{x}{y}} \cdot e^{C}$$

$$y = c \cdot e^{\frac{x}{y}}$$

(5) szétválasztható:

$$y'(x) = (1+y^2(x)) \ln x \qquad x>0$$

$$y' = (1+y^2) \ln x \qquad /:(1+y^2)$$

$$\int \frac{1}{1+y^2}\,dy = \int \ln x\,dx$$

?

$$\operatorname{arctg} y = \int x' \ln x\,dx$$

?

$$\operatorname{arctg} y = x \ln x - \int x \cdot \frac{1}{x}\,dx$$

$$\int \ln x\,dx = x \ln x - x + C \;!$$

$$\operatorname{arctg} y = x \ln x - x + C$$

$$\operatorname{tg}(\operatorname{arctg} y) = \operatorname{tg}(x \ln x - x + C)$$

$$y = \operatorname{tg}(x \ln x - x + C)$$

(7) változóiban homogén:

$$\boxed{y'(x) = f\left(\frac{y(x)}{x}\right) \text{ alakúak}}$$

$$y'(x) = \frac{y(x)+x}{x} \qquad x \neq 0$$

$$y' = \frac{y+x}{x}$$

$$y' = \frac{y}{x} + 1$$

$$u = \frac{y}{x} \qquad y = ux \qquad y' = u'x + ux' = u'x + u$$

$$u'x + \cancel{u} = \cancel{u} + 1$$

$$u'x = 1 \qquad /:x$$

$$u' = \frac{1}{x}$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\int du = \int \frac{1}{x}\,dx$$

$$u = \ln|x| + C \qquad x \neq 0 \qquad C \in \mathbb{R}$$

$$\frac{y}{x} = \ln x + C.$$

$$y = x(\ln x + C)$$

$$\underline{\underline{y = x \ln x + C}}$$

⑧ változóiban homogén:

$$y'(x) = \frac{y(x)+x}{y(x)+3x}$$

$$\boxed{u = \frac{y}{x} \qquad y = ux \qquad y' = u'x + u}$$

$$y' = \frac{\frac{y}{x}+1}{\frac{y}{x}+3} \rightarrow y' = \frac{u+1}{u+3} \rightarrow u'x + u = \frac{u+1}{u+3} \quad /-u$$

$$u'x = \frac{u+1}{u+3} - u$$

$$u'x = \frac{u+1}{u+3} - \frac{u^2+3u}{u+3} = \frac{-u^2-2u+1}{u+3}$$

$$\frac{du}{dx}x = \frac{-u^2-2u+1}{u+3}$$

$$\int \frac{u+3}{-u^2-2u+1}\,du = \int \frac{1}{x}\,dx \qquad /\cdot(-1)$$

$$\int \frac{u+3}{u^2+2u-1}\,du = -\int \frac{1}{x}\,dx$$

$$u^2+2u-1=0$$

$$u_{12} = \frac{-2 \pm \sqrt{4+4}}{2} \quad \sqrt{8}=2\sqrt{2} \quad \begin{cases} \frac{-2+2\sqrt{2}}{2} = \boxed{-1+\sqrt{2}} \\ \frac{-2-2\sqrt{2}}{2} = \boxed{-1-\sqrt{2}} \end{cases}$$

$$\frac{u+3}{u^2+2u-1} = \frac{u+1}{u^2+2u-1} + \frac{2}{u^2+2u+1} = \frac{1}{2}\cdot\frac{2u+2}{u^2+2u-1} + 2\cdot\frac{1}{u^2+2u+1}$$

$$\frac{1}{2}\int \frac{2u+2}{u^2+2u-1}\,du + 2\int \frac{1}{u^2+2u+1}\,du = -\ln|x| + C$$

?

$$\frac{1}{2}\ln|u^2+2u-1| + \frac{1}{\sqrt{2}}\ln|u+1-\sqrt{2}| - \frac{1}{\sqrt{2}}|u+1+\sqrt{2}| = -\ln|x| + C \qquad /\cdot 2$$

$$\ln|u^2+2u-1| + \sqrt{2}\ln|u+1-\sqrt{2}| - \sqrt{2}|u+1+\sqrt{2}| = -2\ln|x| + C$$

$$\ln \frac{|u^2+2u-1|\cdot|u+1-\sqrt{2}|^{\sqrt{2}}}{|u+1+\sqrt{2}|^{\sqrt{2}}} = \ln\frac{1}{x^2} + C$$

$$|u^2+2u-1|\,\frac{|u+1-\sqrt{2}|^{\sqrt{2}}}{|u+1+\sqrt{2}|^{\sqrt{2}}} = \frac{1}{x^2} + e^C$$

?

$$(u+1-\sqrt{2})^{1+\sqrt{2}}(u+1+\sqrt{2})^{1-\sqrt{2}} = C\cdot\frac{1}{x^2}$$

$$\left(\frac{y}{x}+1-\sqrt{2}\right)^{1+\sqrt{2}}\left(\frac{y}{x}+1+\sqrt{2}\right)^{1-\sqrt{2}} = C\cdot\frac{1}{x^2}$$

elsőrendű lineáris

$$y'(x) + y(x)\cdot tg\, x = \sin 2x \qquad x \in \,]0;\tfrac{\pi}{2}[$$

$$y\left(\tfrac{\pi}{4}\right) = 1$$

$$y'(x) = y(x)\cdot tg(x) = 0$$

$$y(x) = c\cdot e^{-\int tg\, x\, dx} = c\cdot e^{\int -\frac{\sin x}{\cos x}dx} = c\cdot e^{\ln \cos x} = c\cdot \cos x$$

$$y_p(x) = c(x)\cos x$$

$$c'(x)\cos x = \sin 2x = 2\sin x \cos x$$

$$c'(x) = 2\sin x$$

$$c(x) = -2\cos x$$

$$y_p(x) = -2\cos^2 x \Rightarrow y(x) = \cos x - 2\cos^2 x \quad ; \quad x \in \,]0;\tfrac{\pi}{2}[$$

$$1 = y\left(\tfrac{\pi}{4}\right) = c\cdot\frac{\sqrt{2}}{2} - 2\cdot\frac{1}{2}$$

$$\frac{4}{\sqrt{2}} = c = 2\sqrt{2}$$

$$y(x) = 2\sqrt{2}\cos x - 2\cos^2 x$$

(9) elsőrendű lineáris:

$$\boxed{y' + f(x)y = g(x) \quad \text{alakú}}$$

1. lépés: $y'(x) - xy(x) = x^3$

$$y' - xy = x^3$$

$$y' - xy = 0$$

$$y' = xy$$

$$\frac{dy}{dx} = xy$$

$$\int \frac{1}{y}\,dy = \int x\,dx$$

$$\ln|y| = \frac{x^2}{2} + c$$

$$|y| = e^{\frac{x^2}{2}+c}$$

$$y = \pm e^{\frac{x^2}{2}}\cdot e^{c}$$

$$y = e^{\frac{x^2}{2}}\cdot C \qquad C\in\mathbb{R}$$

$$y' + f(x)y = 0$$

$$y' = -f(x)y$$

$$\frac{dy}{dx} = -f(x)y$$

$$\int \frac{1}{y}\,dy = -\int f(x)\,dx$$

$$\ln y = -\int -x\,dx$$

$$y = c\cdot e^{\int x\,dx} = c\cdot e^{\frac{x^2}{2}} \qquad c\in\mathbb{R}$$

2. lépés:

$$y_p = c(x)\, e^{\frac{x^2}{2}} \qquad c(x) = ?$$

$$y'_p = c' e^{\frac{x^2}{2}} + c\cdot e^{\frac{x^2}{2}}\cdot \frac{1}{2}\cdot 2x = c'\cdot e^{\frac{x^2}{2}} + c\cdot x\cdot e^{\frac{x^2}{2}}$$

behelyettesítve: $y' - xy = x^3$ -be

$$c'\cdot e^{\frac{x^2}{2}} + \underbrace{c\cdot x\cdot e^{\frac{x^2}{2}} - x\cdot c\cdot e^{\frac{x^2}{2}}}_{0} = x^3$$

$$c'\cdot e^{\frac{x^2}{2}} = x^3$$

$$c' = x^3\cdot e^{-\frac{x^2}{2}}$$

$$\int dc = \int x^3 e^{-\frac{x^2}{2}}\, dx$$

$$c = -\int x^2 (-x)\cdot e^{-\frac{x^2}{2}}\, dx =$$

$$c = -\int x^2 \left(e^{-\frac{x^2}{2}}\right)' dx$$

$$c = -x^2 e^{-\frac{x^2}{2}} + \int 2x e^{-\frac{x^2}{2}}\, dx$$

$$c = -x^2 e^{-\frac{x^2}{2}} + 2\int (-x)\, e^{-\frac{x^2}{2}}\, dx$$

$$c = -x^2\cdot e^{-\frac{x^2}{2}} - 2 e^{-\frac{x^2}{2}} + k$$

$$c = \left(-x^2 - 2\right) e^{-\frac{x^2}{2}} + k$$

$$y_p = (-x^2-2)\underbrace{e^{-\frac{x^2}{2}}\cdot e^{\frac{x^2}{2}}}_{1}, \text{ ha } k=0$$

ált. mo: $\boxed{y = c\, e^{\frac{x^2}{2}} - (-x^2-2) = c\cdot e^{\frac{x^2}{2}} - (x^2+2)}$ $c \in \mathbb{R}$

(9)

$$y' - xy = x^3$$

$$y' = xy + x^3 \Rightarrow f(x) = x, \quad g(x) = x^3 \qquad F = \int x\, dx = \frac{x^2}{2} + C$$

$$\boxed{\begin{aligned} & y' = f(x)y + g(x) \\ & \text{mo: } y = e^{F}\int g\, e^{-F}\, dx \end{aligned}}$$

Parciális: $\int uv' = u\cdot v - \int u'v$

$$y = e^{\frac{x^2}{2}}\cdot \int \underset{u}{x^2}\cdot \underset{v'}{x\, e^{-\frac{x^2}{2}}}\, dx = e^{\frac{x^2}{2}}\cdot\left(-x^2\cdot e^{-\frac{x^2}{2}} - \int x^2\cdot e^{-\frac{x^2}{2}} dx\right) =$$

$$= e^{\frac{x^2}{2}}\cdot\left(-x^2\cdot e^{-\frac{x^2}{2}} - 2\cdot e^{-\frac{x^2}{2}} + C\right) = -x^2 - 2 + c\cdot e^{\frac{x^2}{2}}$$

(10) elsőrendű lin:

$y' = f(x)y + g(x)$

$y = e^{F}\int g e^{-F} dx$

$y'(x) + y = e^{-x}$

$y'(x) = -y + e^{-x}$

$f(x) = -1$ $\qquad F = \int -1\, dx$

$g(x) = e^{-x}$ $\qquad F = -x + C$

$y = e^{-x} \cdot \int e^{-x} \cdot e^{x} dx = e^{-x} \cdot \int 1\, dx = \underline{\underline{e^{-x} \cdot x + C}}$

(11) elsőrendű lin:

$y'(x) + \frac{2}{x} y(x) = 3 \qquad y(1) = 0$

$y' = -\frac{2}{x} y + 3$

$y' = f(x)y + g(x)$

$y = e^{F}\int g e^{-F} dx$

$f(x) = -\frac{2}{x}$ $\qquad F = \int -\frac{2}{x} dx$

$g(x) = 3$ $\qquad F = -2\int \frac{1}{x} dx$

$F = -2 \ln|x| + C$

$y = e^{-2\ln|x|}\int 3 e^{2\ln|x|} dx = e^{\ln|x|^{-2}} 3 \int e^{\ln|x|^{2}} dx = |x|^{-2} \cdot 3 \int |x|^{2} dx = x^{-2} \cdot 3 \cdot \frac{x^3}{2 \cdot 3} = \underline{\underline{x + C}}$

(12) elsőrendű lin:

$y'(x) = y(x) + x$

$y' = f(x)y + g(x)$

$y = e^{F}\int g e^{-F} dx$

$f(x) = 1$ $\qquad F = \int dx$

$g(x) = x$ $\qquad F = x + C$

$\int u v' = uv - \int u' v$

$y = e^{x}\int \underset{u}{x} \underset{v'}{e^{-x}} dx = e^{x} \cdot x \cdot (-e^{-x}) - \int e^{-x} dx =$

$\int e^{-x} = \frac{e^{-x}}{-1} = -e^{-x}$

$y = \cancel{e^{x}} \cdot x \cdot (-\cancel{e^{-x}}) + e^{-x} + C$

$\underline{\underline{y = -x + e^{-x} + C}}$

(11) Bernoulli

$y'(x) + y(x) = -\frac{1}{y(x)}$

$y' + f(x)\,y = g(x)\,y^{\alpha}$

$y' + y = -y^{-1}$

$u' + (1-\alpha) f(x)\, u = g(x)(1-\alpha)$

$u = y^{1-\alpha}$

$y = u^{\frac{1}{1-\alpha}}$

$\alpha = -1 \qquad \underline{1-\alpha} = 1-(-1) = \underline{2}$

$f(x) = 1 \qquad g(x) = -1$

$u = y^2$

$y = u^{\frac{1}{2}}$

$u' + 2 \cdot 1 \cdot u = (-1) \cdot 2$

$u' + 2u = -2$

$u' = -2 - 2u$

$u' = -2(1+u) \quad / : 1+u$

$\frac{1}{1+u} u' = -2$

$\int \frac{1}{1+u}\, dy = -2 \int dx$

$\ln|1+u| = -2x + c$

$1 + u = e^{-2x+c}$

$1 + u = e^{-2x} \cdot C \qquad (c \in \mathbb{R})$

$u = e^{-2x} \cdot c - 1$

$y = (e^{-2x} \cdot c - 1)^{\frac{1}{2}} \Rightarrow \underline{\underline{y = \sqrt{\frac{1}{e^2} \cdot c - 1}}}$

(9) Elsőrendű lin.:

1) $y'(x) + f(x)y(x) = g(x)$ $\qquad$ $f(x) = c(x) \cdot e^{-\int f(x)dx}$

2) $y'(x) + f(x)y(x) = 0$ $\qquad$ $f_p(x) = \left(\int g(x) \cdot e^{\int f(x)dx} dx + c\right) \cdot e^{-\int f(x)dx}$

$y'(x) - x \cdot y(x) = x^3$ $\qquad$ $f(x) = x$ $\qquad$ $g(x) = x^3$

$y' - xy = 0$

$y(x) = c \cdot e^{-\int -x\,dx} = c \cdot e^{\frac{x^2}{2}}$

$$f_p(x) = \left(\int x^3 \cdot e^{\int -x\,dx} dx + c\right) \cdot e^{-\int -x\,dx} = \left(-\int \underset{u}{-x^3} \cdot \underset{v'}{e^{-\frac{x^2}{2}}} + c\right) e^{+\frac{x^2}{2}} =$$

$$\int \underset{u}{-x^3} \cdot \underset{v'}{e^{-\frac{x^2}{2}}} dx = \underset{u \cdot v - \int u'v}{-x^2 \cdot e^{-\frac{x^2}{2}} - 2\int -x \cdot e^{-\frac{x^2}{2}} dx} = \left(-x^2 \cdot e^{-\frac{x^2}{2}} - 2e^{-\frac{x^2}{2}} + c\right)$$

$$= \left(-x^2 \cdot e^{-\frac{x^2}{2}} - 2e^{-\frac{x^2}{2}} + c\right) e^{+\frac{x^2}{2}} = \boxed{-x^2 - 2 + c \cdot e^{\frac{x^2}{2}}}$$

(14) Bernoulli:

$y'(x) - y(x) = -(1+x)\,y^2(x)$

$y' - y = -(1+x)\,y^2$

$\boxed{y' + f(x)\,y = g(x)\,y^{\alpha}}$ alakú

mo: $u'(x) + (1-\alpha) f(x)\,u(x) = g(x)$

$\boxed{\alpha = 2}$ $\boxed{1-\alpha} = 1-2 = \boxed{-1}$

$\boxed{f(x) = -1}$ $\boxed{g(x) = -(1+x)}$

$u = y^{1-\alpha} \Rightarrow \boxed{u = y^{-1}}$

$y = u^{\frac{1}{1-\alpha}} \Rightarrow \boxed{y = u^{-1}}$

$u' + (-1)(-1)u = -(1+x)(-1)$

$\to u' + u = 1+x$

↳ elsőrendű diffe.

homogenizált egyen.

$u' + u = 0$

$u' = -u$

$\frac{1}{u} u' = -1$

$\frac{1}{u} \frac{du}{dx} = -1$

$\int \frac{1}{u}\,du = \int -1\,dx$

$\ln|u| = -x + c$

$|u| = e^{-x+c}$

$|u| = e^{-x} \cdot e^{c} \to |u| = e^{-x} \cdot C$ $(c \in \mathbb{R})$

$u' = \int \underset{v'}{e^{-x}} \cdot \underset{u}{C} = c' \cdot e^{-x} - c \cdot e^{-x}$ $u = c \cdot e^{-x}$

$u' + u = 1+x$

$c' e^{-x} - \cancel{ce^{-x}} + \cancel{ce^{-x}} = 1+x$

$c' e^{-x} = 1+x$

$c' = (1+x)e^{x}$

$\int dc = \int (1+x)e^{x}\,dx$

$c = \int (1+x)e^{x}\,dx \to$ parciális int.

$\int \underset{u}{(1+x)}\underset{v'}{e^{x}}\,dx = (1+x)e^{x} - \int e^{x}\,dx = (1+x)e^{x} - e^{x} + k$

$c = \cancel{e^{x}} + xe^{x} - \cancel{e^{x}} + k \Rightarrow c = xe^{x} + k \to u = c \cdot e^{-x} + x$ $c \in \mathbb{R}$

$y = u^{-1} = (c \cdot e^{-x} + x)^{-1} \Rightarrow$ $\underline{\underline{y = \frac{1}{ce^{-x} + x}}}$ felt: $ce^{-x} + x \neq 0$

$c \neq -xe^{x}$

(15) Bernoulli:

$$y'(x)+y(x)+y^2(x)=0$$

$$\boxed{y'+f(x)y=g(x)y^2 \text{ alakú}}$$

$$u'+(1-\alpha)f(x)u=g(x)(1-\alpha)$$

$$y'+y=-y^2$$

$\alpha=2 \qquad 1-\alpha=-1$

$f(x)=1 \qquad g(x)-1$

$u=y^{1-\alpha} \rightarrow u=y^{-1}$

$y=u^{\frac{1}{1-\alpha}} \rightarrow y=u^{-1}$

$$u'+(-1)\cdot 1u=(-1)(-1)$$

$$u'-u=1 \qquad |+u$$

$$u'=1+u \qquad /:1+u$$

$$\frac{u'}{1+u}=1$$

$$\frac{1}{1+u}u'=1$$

$$\int\frac{1}{1+u}\,du=\int dx$$

$$\ln|1+u|=x+c$$

$$|1+u|=e^{x+c}$$

$$|1+u|=e^{x}\cdot c \qquad (c\in\mathbb{R})$$

$$1+u=\pm c\cdot e^{x}$$

$$\boxed{u=e^{x}\cdot c-1}$$

$$y=(e^{x}\cdot c-1)^{-1} \Rightarrow y=\frac{1}{e^{x}\cdot c-1}$$

(16) Bernoulli:

$$y' + f(x)\,y = g(x)\,y^{\alpha}$$

$$u' + (1-\alpha) f(x)\,u = g(x)(1-\alpha)$$

$$y' - y = x \cdot y^3$$

$\alpha = 3 \qquad 1-\alpha = -2$

$f(x) = -1 \qquad g(x) = x$

$u = y^{1-\alpha} \Rightarrow u = y^{-2}$

$y = u^{\frac{1}{1-\alpha}} \Rightarrow y = u^{-\frac{1}{2}}$

$$u' + (-2)\cdot(-1)\,u = x(-2)$$

$u' + 2u = -2x \quad \rightarrow \quad u' + 2u = 0$

$u' = -2x - 2u \qquad u' = -2u$

$$\int \frac{1}{u}\,du = -2\int dx$$

$$\ln|u| = -2x + c$$

$$|u| = e^{-2x} \cdot c$$

$$(f\cdot g)' = f' \cdot g + f \cdot g'$$

$$u = e^{-2x} \cdot c$$

$$u' = -2 \cdot e^{-2x} \cdot c + c' \cdot e^{-2x}$$

$$u' + 2u = -2x$$

$$\cancel{-2 \cdot e^{-2x} \cdot c} + c' \cdot e^{-2x} \cancel{+ 2e^{-2x} \cdot c} = -2x$$

$$\int \underset{u}{x} \cdot \underset{v'}{e^{2x}}\,dx = x \cdot e^{2x} - \int e^{2x}\,dx = x \cdot e^{2x} - \frac{1}{2}e^{2x} + c$$

$$c' \cdot e^{-2x} = -2x$$

$$c' = -2x \cdot e^{2x} \qquad (e^{2x})' = 2e^{x}$$

?

$$\int dc = -\int x\,(2e^{2x})\,dx \longrightarrow -\int x\,(e^{2x})'\,dx$$

$$c = -\left(x \cdot e^{2x} - \frac{1}{2}e^{2x}\right) + k$$

$$c = -x \cdot e^{2x} + \frac{1}{2}e^{2x} + k$$

$$c = \left(\frac{1}{2} - x\right)e^{2x} + k$$

ált. mego: $u = c \cdot e^{-2x} + \left(\frac{1}{2} - x\right) \underbrace{e^{2x} \cdot e^{-2x}}_{1} = c \cdot e^{-2x} + \frac{1}{2} - x \qquad c \in \mathbb{R}$

$$y = \left(c \cdot e^{-2x} + \frac{1}{2} - x\right)^{-\frac{1}{2}}$$

(17) Ricatti:

$$y'(x) + \frac{1}{x} y(x) + y(x)^2 = \frac{4}{x^2} \qquad v(x) = \frac{2}{x}$$

$$y' + \frac{1}{x} + y^2 = \frac{4}{x^2}$$

$$v(x) = \frac{2}{x} \text{ megoldás}$$

ell: $-\frac{2}{x^2} + \frac{1}{x} \cdot \frac{2}{x} + \frac{4}{x^2} = \frac{4}{x^2}$ ✓ igaz

keressük az összes megoldást.

$$y = u + v \Rightarrow y = u + \frac{2}{x}$$

$$y' = u' - \frac{2}{x^2}$$

Behelyettesítve: $u' - \frac{2}{x^2} + \frac{1}{x}\left(u + \frac{2}{x}\right) + \left(u + \frac{2}{x}\right)^2 = \frac{4}{x^2}$

$$u' - \cancel{\frac{2}{x^2}} + \frac{1}{x} \cdot u + \cancel{\frac{2}{x^2}} + u^2 + \frac{4}{x} u + \cancel{\frac{4}{x^2}} = \cancel{\frac{4}{x^2}}$$

$u' + \frac{5}{x} u = -u^2$ ez már Bernoulli de.

ahol $f(x) = \frac{5}{x}$ $g(x) = -1$, $\alpha = 2$

$$z = u^{1-\alpha} = u^{1-2} = u^{-1} \Rightarrow u = z^{\frac{1}{1-\alpha}} = z^{-1}$$

a megoldást keressük: $z' + (1-\alpha) f(x) \cdot z = g(x) \cdot (1-\alpha)$ alakban

$$z' + (-1) \cdot \frac{5}{x} \cdot z = (-1)(-1)$$

$z' - \frac{5}{x} \cdot z = 1$ ez lin. diffe.

$$z' = \frac{5}{x} z + 1$$

$$y' = f(x) y + g(x)$$

$$y = e^{F} \int g e^{-F} dx$$

$f(x) = \frac{5}{x}$ $\quad F = \int \frac{5}{x} dx$

$g(x) = 1$ $\quad F = 5 \int \frac{1}{x} dx$

$$F = 5 \ln|x|$$

$$y = e^{5\ln|x|} \cdot \int e^{-x} \cdot e^{-5\ln|x|} dx$$

$$y = e^{5\ln|x|} \cdot \int e^{-5\ln|x| - x} dx$$

$$y = e^{5\ln|x|} \cdot e^{-5\ln|x| - x} + C = \cancel{e^{5\ln|x|}} \cdot e^{-5\ln|x|} \cdot e^{-x} = e^{-x} + C$$

1.) $z' - \frac{5}{x} z = 0$

$$z' = \frac{5}{x} \cdot z$$

$$\int \frac{1}{z} dz = \int \frac{5}{x} dx$$

$$\ln|z| = 5 \ln|x| + C$$

$$|z| = |x|^5 \cdot e^C$$

$z = \pm x^5 \cdot e^C$ $\quad z = 0$ mego.

$$z = c \cdot x^5$$

$z_{\text{sub, P}} = c(x) \cdot x^5$

$z' = c'x^5 + 5x^4 \cdot c$

$c' \cdot x^5 + \cancel{5x^4 c - \frac{5}{x} \cdot c \cdot x^5} = 1$

$$c' x^5 = 1$$

$$c' = \frac{1}{x^5}$$

$$\int dc = \int \frac{1}{x^5} dx$$

$z_{\text{sub, P}} = \frac{-x^{-4}}{4} \cdot x^5 = -\frac{x}{4} \Leftarrow \quad c = \frac{x^{-4}}{-4} + \underline{k = 0}$

$\Rightarrow z = c \cdot x^5 - \frac{x}{4}$

$\Rightarrow u = \left(c \cdot x^5 - \frac{x}{4}\right)^{-1}$

$\Rightarrow \underline{\underline{y = \left(c \cdot x^5 - \frac{x}{4}\right)^{-1} + \frac{2}{x} \quad c \in \mathbb{R}}}$

Szétválasztható:

$y'(x) = y(x)$

$y'(x) = x\,y(x)$

$y'(x) = 4x\sqrt{y(x)} \qquad y(1) = 1$

$y'(x) = x\,e^{y(x)}$

$y'(x) = (1 + y^2(x)) \ln x$

Változóiban homogén:

$y'(x) = \dfrac{y(x)}{y(x) + x}$

$y'(x) = \dfrac{y(x) + x}{x}$

$y'(x) = \dfrac{y(x) + x}{y(x) + 3x}$

Elsőrendű lineáris:

$y'(x) - x\,y(x) = x^3$

$y'(x) + y(x) = e^{-x}$

$y'(x) + \frac{2}{x} y(x) = 3, \quad y(1) = 0$

$y'(x) = y(x) + x$

Bernoulli:

$y'(x) + y(x) = -\dfrac{1}{y(x)}$

$y'(x) - y(x) = -(1+x)\,y(x)^2$

$y'(x) + y(x) + y(x)^2 = 0$

$y'(x) - y(x) = x \cdot y(x)^3$

Ricatti:

$y'(x) + \frac{1}{x} y(x) + y(x)^2 = \dfrac{4}{x^2} \qquad v(x) = \dfrac{2}{x}$

$y'(x) + \frac{1}{3} y(x)^2 + \frac{2}{3} \cdot \dfrac{1}{x^2} = 0 \qquad v(x) = \dfrac{1}{x}$

$y'(x) + 2\,y(x)\,e^x - y(x)^2 = e^{2x} - e^x \qquad v(x) = e^x$

Egzakt:

$x^2 + y(x)^2 + 2x\,y(x)\,y'(x) = 0$

$2x\,y(x) + 3y(x)^2 + (x^2 + 6x\,y(x) - 2y(x))\,y'(x) = 0$

$2x + y(x) + (x - 2y(x))\,y'(x) = 0$

Hiányos másodfokú:

$2x\,y''(x) + y'(x) = 0$

$2y(x)\,y'(x) = y''(x)$

$y''(x) = \dfrac{y'(x)^2}{y(x)}$

$y''(x) = e^{y(x)}$

$$\left(\log_a x\right)' = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\int \frac{1}{\ln x} \cdot \frac{1}{1} = \log_x 1 = a$$