$y'(x) = 4x\sqrt{y(x)}$ $\qquad$ $y(1) = 1$

$\frac{dy}{dx} = 4x(y(x))^{\frac{1}{2}}$

$\int y^{-\frac{1}{2}}\, dy = \int 4x\, dx$

$\frac{y^{\frac{1}{2}}}{\frac{1}{2}} = 4\frac{x^2}{2} + c_1$

$\cancel{2}y^{\frac{1}{2}} = \cancel{2}x^2 + c$

$y = (\cancel{2}x^2 + c)^2$

$y'(x) = xe^{y(x)}$

$\frac{dy}{dx} = xe^{y}$

$\int e^{-y}\, dy = \int x\, dx$

$-e^{-y} = \frac{x^2}{2} + c_1$

$e^{-y} = -\frac{x^2}{2} + c_2$

$-y = \ln\left(-\frac{x^2}{2} + c_2\right)$

$\boxed{y = -\ln\left(-\frac{x^2}{2} + c_2\right)}$

$$y'(x) = y(x)$$

$$\frac{dy}{dx} = y(x)$$

$$dy = y(x)\,dx$$

$$\int \frac{1}{y}\,dy = \int 1\,dx$$

$$\ln y = x + C_1$$

$$\boxed{y = e^x \cdot C}$$

$$y'(x) = x\,y(x)$$

$$\frac{dy}{dx} = x\,y(x)$$

$$\int \frac{1}{y}\,dy = \int x\,dx$$

$$\ln y = \frac{x^2}{2} + C_1$$

$$y = e^{\frac{x^2}{2} + C_1}$$

$$\boxed{y = e^{\frac{x^2}{2}} \cdot C}$$

(1) $y'(x) + y(x) = 4 \cdot e^{x}$

$y'(x) + y(x) = 0$

$\frac{dy}{dx} = -y$

$\int \frac{1}{y}\, dy = -\int 1\, dx$

$\ln y = -x + c_1$

$y_h = c \cdot e^{-x}$

ez a homogén egyenlet megoldása

$y_0 = c(x) \cdot e^{-x}$

$y' = c'(x) \cdot e^{-x} - c(x) \cdot e^{-x}$

visszahelyettesítés:

$c'(x) \cdot e^{-x} \underbrace{- c(x) \cdot e^{-x} + c(x) \cdot e^{-x}}_{0} = 4 \cdot e^{x}$

$c'(x) \cdot e^{-x} = 4 \cdot e^{x}$

$c'(x) = 4 \cdot e^{x} \cdot e^{x}$

$c'(x) = 4 \cdot e^{2x}$

$c(x) = \cancel{4} \cdot \frac{1}{\cancel{2}} \cdot e^{2x} = 2 \cdot e^{2x}$

$y_0 = 2e^{2x} \cdot e^{-x} = 2e^{x}$

$\boxed{y = y_h + y_0 = c \cdot e^{-x} + 2 \cdot e^{x}}$

(2) $y'(x) = x \cdot \sqrt[5]{y(x)}$

$y'(x) = x \left(y(x)\right)^{\frac{1}{5}}$

$\frac{dy}{dx} = x \cdot y^{\frac{1}{5}}$

$\int y^{-\frac{1}{5}}\, dy = \int x \cdot dx$

$\frac{y^{-\frac{1}{5}+1}}{-\frac{1}{5}+1} = \frac{x^2}{2} + c_1$

$\frac{y^{\frac{4}{5}}}{\frac{4}{5}} = \frac{x^2}{2} + c_1$

$y^{\frac{4}{5}} = \frac{\cancel{4}^{2}}{5} \cdot \frac{x^2}{\cancel{2}} + c$

$\boxed{y = \left(\frac{2x^2}{5} + c\right)^{-\frac{4}{5}}}$

Az elv minden esetben jó, de a számolásban lehet hiba! Jó munkát! Szia!

$$u'(x) = \frac{1}{x}u(x) + u(x)^2$$

$$y'(x) + f(x)\cdot y(x) = g(x)\cdot y(x)^{\alpha}$$

$$u'(x) - \frac{1}{x}u(x) = u(x)^2$$

$z(x) = u^{-1}$ $\quad \alpha = 2$

$f(x) = -\frac{1}{x}$

$g(x) = 1$

$$z'(x) + (1-\alpha)\cdot f(x)\cdot z(x) = g(x)(1-\alpha)$$

$$z'(x) + (-1)\left(-\frac{1}{x}\right)\cdot z(x) = 1\cdot(-1)$$

$$z'(x) + \frac{1}{x}\cdot z(x) = -1$$

Ez elsőrendű lin

$$z'(x) + \frac{1}{x}z(x) = 0$$

$$z'(x) = -\frac{1}{x}z(x)$$

$$\frac{dz}{dx} = -\frac{1}{x}\cdot z(x)$$

$$\int \frac{1}{z}\,dz = -\int \frac{1}{x}\,dx$$

$$\ln z = -\ln x + c_1$$

$$z_h = c\cdot e^{-\ln x} = c\left(e^{\ln x}\right)^{-1} = c\cdot\frac{1}{x}$$

$$z_0 = c(x)\cdot\frac{1}{x}$$

$$z_0' = c'(x)\cdot\frac{1}{x} + c(x)(-1)\cdot x^{-2} = c'(x)\cdot\frac{1}{x} - c(x)\cdot\frac{1}{x^2}$$

$$c'(x)\cdot\frac{1}{x} \cancel{- c(x)\cdot\frac{1}{x^2} + \frac{1}{x}\cdot c(x)\frac{1}{x}} = -1$$

$$c'(x)\cdot\frac{1}{x} = -1$$

$$c'(x) = -x$$

$$c(x) = -\frac{x^2}{2} \Rightarrow z_0 = -\frac{x^2}{2}\cdot\frac{1}{x} = -\frac{x}{2}$$

$$z = z_h + z_0 = c\frac{1}{x} - \frac{x}{2}$$

$$\Downarrow$$

$$u = \frac{1}{z} = \frac{1}{\frac{c}{x} - \frac{x}{2}}$$

$$\Downarrow$$

$$y = u + v = \boxed{\frac{1}{\frac{c}{x} - \frac{x}{2}} + \frac{1}{x}}$$

(1) $y'(x) + y(x) = 2 \cdot e^{x}$

$y'(x) + y(x) = 0$

$\frac{dy}{dx} = -y(x)$

$\int \frac{1}{y}\, dy = -\int 1\, dx$

$\ln y = -x + c_1$

$y_h = c \cdot e^{-x}$

$y_0 = c(x) \cdot e^{-x}$

$y_0' = c'(x) \cdot e^{-x} - c(x) \cdot e^{-x}$

$c'(x) \cdot e^{-x} \underbrace{- c(x) \cdot e^{-x} + c(x) \cdot e^{-x}}_{0} = 2 \cdot e^{x}$

$c'(x) \cdot e^{-x} = 2 \cdot e^{x}$

$c'(x) = 2 \cdot e^{2x}$

$c(x) = 2 \cdot \frac{1}{2} \cdot e^{2x} = e^{2x}$ $\qquad y_0 = e^{2x} \cdot e^{-x} = e^{x}$

$\boxed{y = y_h + y_0 = c \cdot e^{-x} + e^{x}}$

(2) $y'(x) = x \cdot \sqrt[7]{y(x)}$

$y'(x) = x \cdot y^{\frac{1}{7}}$

$\frac{dy}{dx} = x \cdot y^{\frac{1}{7}}$

$\int y^{-\frac{1}{7}}\, dy = \int x\, dx$

$\frac{y^{-\frac{1}{7}+1}}{-\frac{1}{7}+1} = \frac{x^2}{2} + c_1$

$\frac{y^{+\frac{6}{7}}}{\frac{6}{7}} = \frac{x^2}{2} + c_1$

$y^{\frac{6}{7}} = \frac{\cancel{6}^{3}}{7} \cdot \frac{x^2}{\cancel{2}} + c$

$y^{\frac{6}{7}} = \frac{3x^2}{7} + c$

$\boxed{y = \left(\frac{3x^2}{7} + c\right)^{-\frac{6}{7}}}$

(5) $y'(x) + \frac{1}{x} y(x) + y(x)^2 = \frac{1}{x^2}$ $\qquad v(x) = \frac{1}{x}$ $\qquad y = u + v$

Alapképlet: $y'(x) = f(x) \cdot y(x)^2 + g(x) \cdot y(x) + h(x)$

helyes sorrend
$y'(x) = -y(x)^2 - \frac{1}{x} y(x) + \frac{1}{x^2}$ $\quad \Rightarrow \quad f(x) = 1$, $g(x) = -\frac{1}{x}$, $h(x) = \frac{1}{x^2}$

$u'(x) = \left(g(x) + 2v(x) \cdot f(x)\right) u(x) + f(x) \cdot u(x)^2$

$u'(x) = \left(-\frac{1}{x} + 2\frac{1}{x} \cdot 1\right) u(x) + 1 \cdot u(x)^2$

$u'(x) = \frac{1}{x} u(x) + 1 \cdot u(x)^2$ $\qquad$ Ez Bernoulli