Hf.:

$$\begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 & 0 \\ 3 & 6 & 1 & 0 & 0 & 0 \\ 4 & 9 & 14 & 1 & 1 & 1 \\ 5 & 12 & 24 & 1 & 5 & 9 \\ 9 & 24 & 28 & 1 & 25 & 81 \end{vmatrix} = \underset{\substack{1\ \ 1\ \ 1 \\ 2\ \ 3\ \ 4}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 1 \end{vmatrix}} \cdot \underset{\substack{1\ \ 1\ \ 1 \\ 1\ \ 5\ \ 9}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 5 & 9 \\ 1 & 25 & 81 \end{vmatrix}} = -8 \cdot 128 = -1024$$

11.17-én ZH az eddigiekből!!!

5. gyakorlat

Hf. mego.

$$\begin{vmatrix} -4 & -1 & 2 & 3 & 1 \\ 2 & 3 & 0 & 4 & -1 \\ 0 & -3 & 0 & 5 & 3 \\ -1 & -1 & 0 & 1 & -1 \\ 0 & -4 & -2 & -1 & -2 \end{vmatrix} \overset{3o.}{=} 2 \begin{vmatrix} 2 & 3 & 4 & -1 \\ 0 & -3 & 5 & 3 \\ -1 & -1 & 1 & -1 \\ 0 & -4 & -1 & -2 \end{vmatrix} - 2 \begin{vmatrix} -4 & -1 & 3 & 1 \\ 2 & 3 & 4 & -1 \\ 0 & 3 & 5 & 3 \\ -1 & -1 & 1 & 1 \end{vmatrix} = 2 \begin{vmatrix} 0 & 1 & 6 & -3 \\ 0 & -3 & 5 & 3 \\ -1 & -1 & 1 & -1 \\ 0 & -4 & -1 & -2 \end{vmatrix} - 2 \begin{vmatrix} 0 & 3 & -1 & 5 \\ 0 & 1 & 6 & -3 \\ 0 & -3 & 5 & 3 \\ -1 & -1 & 1 & 1 \end{vmatrix} =$$

$$= 2(-1) \begin{vmatrix} 1 & 6 & -3 \\ -3 & 5 & 3 \\ -4 & -1 & -2 \end{vmatrix} - 2 \begin{vmatrix} 3 & -1 & 5 \\ 1 & 6 & -3 \\ -3 & 5 & 3 \end{vmatrix} = \underline{\underline{-48}}$$

$$\begin{array}{r} \rightarrow \\ \rightarrow \\ \\ \\ \end{array} \begin{vmatrix} 3 & -2 & 4 & 4 \\ 1 & 5 & -2 & 2 \\ 4 & 1 & 3 & -5 \\ 7 & 6 & 2 & 1 \end{vmatrix} = (-1)^{1+2+1+2} \begin{vmatrix} 3 & -2 \\ 1 & 5 \end{vmatrix} \begin{vmatrix} 3 & -5 \\ 2 & 1 \end{vmatrix} + (-1)^{\overset{\text{sor sor o. o}}{1+2+1+3}} \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} \begin{vmatrix} 1 & -5 \\ 6 & 1 \end{vmatrix} + (-1)^{1+2+1+4} \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 3 \\ 6 & 2 \end{vmatrix} + (-1)^{1+2+2+3} \begin{vmatrix} -2 & 4 \\ 5 & -2 \end{vmatrix} \begin{vmatrix} 4 & -5 \\ 7 & 1 \end{vmatrix} + (-1)^{1+2+2+4} \begin{vmatrix} -2 & 4 \\ 5 & 2 \end{vmatrix} \begin{vmatrix} 4 & 3 \\ 7 & 2 \end{vmatrix} + (-1)^{1+2+3+4} \begin{vmatrix} 4 & 4 \\ -2 & 2 \end{vmatrix} \begin{vmatrix} 4 & 1 \\ 7 & 6 \end{vmatrix}$$

$$\begin{array}{cccc} & \downarrow & & \downarrow \end{array}$$
$$\begin{vmatrix} 1 & 0 & 2 & -1 \\ -1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 0 \\ -1 & 1 & 2 & 1 \end{vmatrix} = \underbrace{(-1)^{1+2+2+4} \begin{vmatrix} 0 & -1 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix}}_{0} + (-1)^{1+3+2+4} \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix} + (-1)^{1+4+6} \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} + (-1)^{2+3+6} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} + (-1)^{2+4+6} \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} + (-1)^{3+4+6} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix}$$

Másféleképpen:

$$\begin{array}{c}\rightarrow\\\rightarrow\\ \\ \\\end{array}\begin{vmatrix}1&0&2&-1\\-1&0&1&1\\2&1&3&0\\-1&1&2&1\end{vmatrix}=\underbrace{(-1)^{1+2+1+2}\begin{vmatrix}1&0\\-1&0\end{vmatrix}\begin{vmatrix}3&0\\2&1\end{vmatrix}}_{0}+(-1)^{1+2+1+3}\begin{vmatrix}1&2\\-1&1\end{vmatrix}\begin{vmatrix}1&0\\1&1\end{vmatrix}+(-1)^{1+2+1+4}\begin{vmatrix}1&-1\\-1&1\end{vmatrix}\begin{vmatrix}1&3\\1&2\end{vmatrix}+$$

$$+\underbrace{(-1)^{1+2+2+3}\begin{vmatrix}0&2\\0&1\end{vmatrix}\begin{vmatrix}2&0\\-1&1\end{vmatrix}}_{0}+\underbrace{(-1)^{1+2+2+4}\begin{vmatrix}0&-1\\0&1\end{vmatrix}\begin{vmatrix}2&3\\-1&2\end{vmatrix}}_{0}+(-1)^{1+2+3+4}\begin{vmatrix}2&-1\\1&1\end{vmatrix}\begin{vmatrix}2&1\\-1&1\end{vmatrix}=\underline{\underline{6}}$$

$$\begin{array}{c} \\\rightarrow\\ \\\rightarrow\\ \\\end{array}\begin{vmatrix}1&0&-2&0&1\\0&-3&0&-1&0\\1&0&7&0&2\\0&-1&0&3&0\\1&0&1&0&1\end{vmatrix}=(-1)^{2+4+2+4}\begin{vmatrix}3&-1\\-1&3\end{vmatrix}\begin{vmatrix}1&-2&1\\1&7&2\\1&1&1\end{vmatrix}$$

Másféleképpen:

$$\begin{vmatrix}1&0&-2&0&1\\0&-3&0&-1&0\\1&0&7&0&2\\0&-1&0&3&0\\1&0&1&0&1\end{vmatrix}\begin{array}{l} \\ \\+(-1)\cdot 1.\text{sor}\\+\frac{1}{3}\cdot 2.\text{sor}\\+(-1)\cdot 1.\text{sor}\end{array}=\begin{vmatrix}1&0&-2&0&1\\0&3&0&-1&0\\0&0&9&0&1\\0&0&0&\frac{8}{3}&0\\0&0&3&0&0\end{vmatrix}\begin{array}{l} \\ \\ \\ \\+3.\text{sor}\cdot\left(\frac{-1}{3}\right)\end{array}=\begin{vmatrix}1&0&-2&0&1\\0&3&0&-1&0\\0&0&9&0&1\\0&0&0&\frac{8}{3}&0\\0&0&0&0&-\frac{1}{3}\end{vmatrix}=1\cdot3\cdot9\cdot\frac{8}{3}\cdot\left(-\frac{1}{3}\right)=-24$$

Hf: tel.

$$\begin{array}{c} \\\rightarrow\\ \\\rightarrow\\ \\\rightarrow\end{array}\begin{vmatrix}1&3&0&4&-3&1\\2&0&4&0&0&3\\1&-3&2&1&4&-2\\3&0&-3&0&0&3\\2&3&-2&4&-2&1\\4&0&-3&0&0&3\end{vmatrix}=(-1)^{\overbrace{2+4+6}^{12}+1+2+3}\begin{vmatrix}2&0&4\\3&0&-3\\4&0&-3\end{vmatrix}\begin{vmatrix}4&-3&1\\1&4&-2\\4&-2&1\end{vmatrix}+(-1)^{12+1+2+4}\begin{vmatrix}2&0&0\\3&0&0\\4&0&0\end{vmatrix}\begin{vmatrix}0&-3&1\\2&4&-2\\-2&-2&1\end{vmatrix}+(-1)^{12+1+2+5}\begin{vmatrix}2&0&0\\3&0&0\\4&0&0\end{vmatrix}\begin{vmatrix}0&4&1\\2&1&-2\\-2&4&1\end{vmatrix}+$$

$$+(-1)^{12+1+2+6}\begin{vmatrix}2&0&3\\3&0&3\\4&0&3\end{vmatrix}\begin{vmatrix}0&4&-3\\2&1&4\\-2&4&-2\end{vmatrix}+(-1)^{12+2+3+4}\begin{vmatrix}0&4&0\\0&-3&0\\0&-3&0\end{vmatrix}\begin{vmatrix}1&-3&1\\1&4&-2\\2&-2&1\end{vmatrix}+(-1)^{12+2+3+5}\begin{vmatrix}0&4&0\\0&-3&0\\0&-3&0\end{vmatrix}\begin{vmatrix}1&4&1\\1&1&-2\\2&4&1\end{vmatrix}+$$

$$+(-1)^{12+2+3+6}\begin{vmatrix}0&4&3\\0&-3&3\\0&-3&3\end{vmatrix}\begin{vmatrix}1&4&-3\\1&1&4\\2&4&-2\end{vmatrix}+(-1)^{12+3+4+5}\begin{vmatrix}4&0&0\\-3&0&0\\-3&0&0\end{vmatrix}\begin{vmatrix}1&3&1\\1&-3&-2\\2&3&1\end{vmatrix}+(-1)^{12+3+4+6}\begin{vmatrix}4&0&3\\-3&0&3\\-3&0&3\end{vmatrix}\begin{vmatrix}1&3&-3\\1&-3&4\\2&3&-2\end{vmatrix}+(-1)^{12+4+5+6}\begin{vmatrix}0&0&3\\0&0&3\\0&0&3\end{vmatrix}\begin{vmatrix}1&3&0\\1&-3&2\\2&3&-2\end{vmatrix}$$

$\hookrightarrow$ folyt!

Hf.:

$$\begin{array}{r} \\ \rightarrow \\ \\ \rightarrow \\ \\ \rightarrow \end{array}\begin{vmatrix} 1 & 3 & 0 & 4 & -3 & 1 \\ \cancel{2} & \cancel{0} & \cancel{4} & \cancel{0} & \cancel{0} & \cancel{3} \\ 1 & -3 & 2 & 1 & 4 & -2 \\ \cancel{3} & \cancel{0} & \cancel{-3} & \cancel{0} & \cancel{0} & \cancel{3} \\ 2 & 3 & -2 & 4 & -2 & 1 \\ \cancel{4} & \cancel{0} & \cancel{-3} & \cancel{0} & \cancel{0} & \cancel{3} \end{vmatrix} = (-1)^{12+1+2+3}\begin{vmatrix} 2 & 0 & 4 \\ 3 & 0 & -3 \\ 4 & 0 & -3 \end{vmatrix}\begin{vmatrix} 4 & -3 & 1 \\ 1 & 4 & -2 \\ 4 & -2 & 1 \end{vmatrix} + (-1)^{12+1+2+4}\begin{vmatrix} 2 & 0 & 0 \\ 3 & 0 & 0 \\ 4 & 0 & 0 \end{vmatrix}\begin{vmatrix} 0 & -3 & 1 \\ 2 & 4 & -2 \\ -2 & -2 & 1 \end{vmatrix} + (-1)^{12+1+2+5}\begin{vmatrix} 2 & 0 & 0 \\ 3 & 0 & 0 \\ 4 & 0 & 0 \end{vmatrix}\begin{vmatrix} 0 & 4 & 1 \\ 2 & 1 & -2 \\ -2 & 4 & 1 \end{vmatrix} +$$

$$+(-1)^{12+1+2+6}\begin{vmatrix} 2 & 0 & 3 \\ 3 & 0 & 3 \\ 4 & 0 & 3 \end{vmatrix}\begin{vmatrix} 0 & 4 & -3 \\ 2 & 1 & 4 \\ -2 & 4 & -2 \end{vmatrix} + (-1)^{12+1+3+4}\begin{vmatrix} 2 & 4 & 0 \\ 3 & -3 & 0 \\ 4 & -3 & 0 \end{vmatrix}\begin{vmatrix} 3 & -3 & 1 \\ -3 & 4 & -2 \\ 3 & -2 & 1 \end{vmatrix} + (-1)^{12+1+3+5}\begin{vmatrix} 2 & 4 & 0 \\ 3 & -3 & 0 \\ 4 & -3 & 0 \end{vmatrix}\begin{vmatrix} 3 & 4 & 1 \\ -3 & 1 & -2 \\ 3 & 4 & 1 \end{vmatrix} + (-1)^{12+1+3+6}\begin{vmatrix} 2 & 4 & 3 \\ 3 & -3 & 3 \\ 4 & -3 & 3 \end{vmatrix}\begin{vmatrix} 3 & 4 & -3 \\ -3 & 1 & 4 \\ 3 & 4 & -2 \end{vmatrix} +$$

$$+(-1)^{12+1+4+5}\begin{vmatrix} 2 & 0 & 0 \\ 3 & 0 & 0 \\ 4 & 0 & 0 \end{vmatrix}\begin{vmatrix} 3 & 0 & 1 \\ -3 & 2 & -2 \\ 3 & -2 & 1 \end{vmatrix} + (-1)^{12+1+4+6}\begin{vmatrix} 2 & 0 & 3 \\ 3 & 0 & 3 \\ 4 & 0 & 3 \end{vmatrix}\begin{vmatrix} 3 & 0 & -3 \\ -3 & 2 & 4 \\ 3 & 2 & -2 \end{vmatrix} + (-1)^{12+2+3+4}\begin{vmatrix} 0 & 4 & 0 \\ 0 & -3 & 0 \\ 0 & -3 & 0 \end{vmatrix}\begin{vmatrix} 1 & -3 & 1 \\ 1 & 4 & -2 \\ 2 & -2 & 1 \end{vmatrix} + (-1)^{12+2+3+5}\begin{vmatrix} 0 & 4 & 0 \\ 0 & -3 & 0 \\ 0 & -3 & 0 \end{vmatrix}\begin{vmatrix} 1 & 4 & 1 \\ 1 & 1 & -2 \\ 2 & 4 & 1 \end{vmatrix} +$$

$$+(-1)^{12+2+3+6}\begin{vmatrix} 0 & 4 & 3 \\ 0 & -3 & 3 \\ 0 & -3 & 3 \end{vmatrix}\begin{vmatrix} 1 & 4 & -3 \\ 1 & 1 & 4 \\ 2 & 4 & -2 \end{vmatrix} + (-1)^{12+2+4+5}\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}\begin{vmatrix} 1 & 0 & 1 \\ 1 & 2 & -2 \\ 2 & -2 & 1 \end{vmatrix} + (-1)^{12+2+4+6}\begin{vmatrix} 0 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 0 & 3 \end{vmatrix}\begin{vmatrix} 1 & 0 & -3 \\ 1 & 2 & 4 \\ 2 & -2 & -2 \end{vmatrix} + (-1)^{12+3+4+5}\begin{vmatrix} 4 & 0 & 0 \\ -3 & 0 & 0 \\ -3 & 0 & 0 \end{vmatrix}\begin{vmatrix} 1 & 3 & 1 \\ 1 & -3 & -2 \\ 2 & 3 & 1 \end{vmatrix} +$$

$$+(-1)^{12+3+4+6}\begin{vmatrix} 4 & 0 & 3 \\ -3 & 0 & 3 \\ -3 & 0 & 3 \end{vmatrix}\begin{vmatrix} 1 & 3 & -3 \\ 1 & -3 & 4 \\ 2 & 3 & -2 \end{vmatrix} + (-1)^{12+4+5+6}\begin{vmatrix} 0 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 0 & 3 \end{vmatrix}\begin{vmatrix} 1 & 3 & 0 \\ 1 & -3 & 2 \\ 2 & 3 & -2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 & 0 \\ 3 & 6 & 1 & 0 & 0 & 0 \\ 4 & 9 & 14 & 1 & 1 & 1 \\ 5 & 12 & 24 & 1 & 5 & 9 \\ 9 & 24 & 38 & 1 & 25 & 81 \end{vmatrix} = (-1)^{6+1+2+3}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 1 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 5 & 9 \\ 1 & 25 & 81 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 & 1 \\ 0 & 1 & 1 & 4 & 1 \\ 0 & 1 & 1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & -2 & 2 & 0 & 1 \\ 0 & 0 & -3 & 3 & 1 \\ 0 & 0 & 0 & -4 & 5 \end{vmatrix} = \underset{(2_o \cdot -1) - 3_o}{\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & -2 & 2 & 0 & 1 \\ 0 & 0 & -3 & 3 & 1 \\ 0 & 0 & 0 & -4 & 5 \end{vmatrix}} = \underset{(3_o \cdot -1) - 4_o}{\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & -3 & 3 & 1 \\ 0 & 0 & 0 & 4 & 5 \end{vmatrix}} =$$

$$= \underset{(3\,\text{sor} \cdot -2) + 5\,\text{sor}}{\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & -4 & 4 & 5 \end{vmatrix}} = \underset{4\,\text{sor} + (5\,\text{sor}(-\frac{9}{4}))}{\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 4 & 3 \end{vmatrix}} = \begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & \frac{3}{12} \end{vmatrix}$$

Nem jó!

$$\begin{vmatrix} 2&1&1&1&1\\ 1&2&1&1&1\\ 1&1&3&1&1\\ 1&1&1&4&1\\ 1&1&1&1&5 \end{vmatrix} \cdot\left(-\tfrac{1}{2}\right)+2.\text{s.} = \begin{vmatrix} 2&1&1&1&1\\ 0&3/2&1/2&1/2&1/2\\ 0&1/2&5/2&1/2&1/2\\ 0&1/2&1/2&7/2&1/2\\ 0&1/2&1/2&1/2&9/2 \end{vmatrix} \cdot\left(-\tfrac{1}{3}\right)+3.\text{sor} = \begin{vmatrix} 2&1&1&1&1\\ 0&3/2&1/2&1/2&1/2\\ 0&0&14/6&2/6&2/6\\ 0&0&2/6&10/3&2/6\\ 0&0&2/6&2/6&13/3 \end{vmatrix} = \begin{vmatrix} 2&1&1&1&1\\ 0&3/2&1/2&1/2&1/2\\ 0&0&7/3&2/6&2/6\\ 0&0&0&23/7&2/7\\ 0&0&0&2/7&30/7 \end{vmatrix} \cdot\left(-\tfrac{2}{23}\right) =$$

$$= \begin{vmatrix} 2&1&1&1&1\\ 0&3/2&1/2&1/2&1/2\\ 0&0&7/3&1/3&1/3\\ 0&0&0&23/7&2/7\\ 0&0&0&0&98/23 \end{vmatrix} = 2\cdot\frac{3}{\cancel{2}}\cdot\frac{\cancel{7}}{\cancel{3}}\cdot\frac{\cancel{23}}{\cancel{7}}\cdot\frac{98}{\cancel{23}} = \underline{\underline{98}}$$

6. gyakorlat

ZH-ban: ami gyakorlaton volt + definíciók. Számológép!!!

Mátrix rangja:

$$\begin{pmatrix} 1&2&3\\ 2&3&4\\ 3&5&7 \end{pmatrix}$$

$$\begin{vmatrix} 1&2&3\\ 2&3&4\\ 3&5&7 \end{vmatrix} = 21+24+30-27-20-28 = 0 \qquad \begin{vmatrix} 1&2\\ 2&3 \end{vmatrix} = -1 \neq 0$$

$$\underline{\underline{\varrho(A)=2}}$$

$$\begin{vmatrix} 1&2&-1&2\\ 2&5&-2&3\\ 1&2&-1&-2\\ 0&5&0&3 \end{vmatrix} = 0$$ (kifejtési tétel a 4. sor alapján)

$$\begin{vmatrix} 1&2&-1\\ 2&5&-2\\ 1&2&-1 \end{vmatrix} = 0 \qquad \begin{vmatrix} 1&2\\ 2&5 \end{vmatrix} = 1 \neq 0$$

$$\begin{vmatrix} 2&-1&2\\ 5&-2&3\\ 2&-1&-2 \end{vmatrix} = -4 \qquad \underline{\underline{\varrho(A)=3}}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$$

- sorok cseréje
- sor szorzása egy $\lambda \neq 0$ számmal és
+ hozzáadása egy másik sorhoz
- olyan sor elhagyása, amely előáll $\forall$ sor lineáris kombinációjaként

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix} \begin{matrix} \cdot(-2) \\ \cdot(-3) \\ \ \end{matrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ \cancel{0} & \cancel{-1} & \cancel{-2} \end{pmatrix} \sim$$ Ahány sora marad, annyi a rangja:

$$\underline{\underline{\rho(A) = 2}}$$

—— o ——

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ \cancel{0} & \cancel{-8} & \cancel{-16} & \cancel{-24} \\ \cancel{0} & \cancel{-12} & \cancel{-24} & \cancel{-36} \end{pmatrix} \Rightarrow \underline{\underline{\rho(A) = 2}}$$

$$\begin{pmatrix} 1 & \lambda & -1 & 2 \\ 2 & -1 & \lambda & 5 \\ 1 & 10 & -6 & 1 \end{pmatrix} \cdot(-2) \sim \begin{pmatrix} 1 & \lambda & -1 & 2 \\ 0 & -2\lambda-1 & 2+\lambda & 1 \\ 0 & 10-\lambda & -5 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -1 & \lambda \\ 0 & 1 & 2+\lambda & -2\lambda-1 \\ 0 & -1 & -5 & 10-\lambda \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -1 & \lambda \\ 0 & 1 & \lambda+2 & -2\lambda-1 \\ 0 & 0 & \lambda-3 & -3\lambda+9 \end{pmatrix}$$

- ha $\underbrace{\lambda - 3 \neq 0 \quad \text{és} \quad -3\lambda + 9 \neq 0}$

$$\lambda \neq 3 \Rightarrow \rho(A) = 3$$

- ha $\lambda = 3 \Rightarrow \rho(A) = 2$.

(ZH-ban olyan lesz, amiben nincs ismeretlen.)

Inverz:

$$\begin{pmatrix} {}^{+}1 & {}^{-}2 & {}^{+}3 \\ {}^{-}2 & {}^{+}3 & {}^{-}2 \\ {}^{+}3 & {}^{-}3 & {}^{+}4 \end{pmatrix} = A \qquad A^{-1} = \frac{\text{adj} A}{\det A} \qquad \text{adj} A = \begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & & \dots & A_{n2} \\ A_{1n} & & \dots & A_{nn} \end{pmatrix}$$

$A_{11} = \begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} = 6$

$A_{12} = -\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -2$

$A_{13} = \begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} = -3$

$A_{21} = -\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 1$

$A_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} = -5$

$A_{23} = -\begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = +3$

$A_{31} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -5$

$A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = +4$

$A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1$

$$\text{adj}\,A = \begin{pmatrix} 6 & 1 & -5 \\ -2 & -5 & 4 \\ -3 & 3 & -1 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{vmatrix} = -7 \qquad \det A = -7$$

$$A^{-1} = \frac{\text{adj}\,A}{\det A} = \frac{1}{\det A} \cdot \text{adj}\,A$$

$$A^{-1} = -\frac{1}{7} \cdot \begin{pmatrix} 6 & 1 & -5 \\ -2 & -5 & 4 \\ -3 & 3 & -1 \end{pmatrix} = \begin{pmatrix} -\frac{6}{7} & -\frac{1}{7} & \frac{5}{7} \\ \frac{2}{7} & 5/7 & -4/7 \\ 3/7 & -3/7 & 1/7 \end{pmatrix}$$

Hf.: ellenőrizni, hogy $A \cdot A^{-1} \overset{?}{=} E$

Csak kvadratikus mátrixnak van inverze!

$$\begin{pmatrix} 5 & -4 \\ -8 & 6 \end{pmatrix} \qquad \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T = \text{adj}\,A$$

$A_{11} = 6$
$A_{12} = +8$
$A_{21} = +4$
$A_{22} = 5$

$$\text{adj}\,A = \begin{pmatrix} 6 & 4 \\ 8 & 5 \end{pmatrix}$$

$$\det A = -2$$

$$A^{-1} = -\frac{1}{2} \cdot \begin{pmatrix} 6 & 4 \\ 8 & 5 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ -4 & -5/2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 2 \\ 0 & 3 & -1 \\ 4 & 1 & 1 \end{pmatrix} \begin{matrix} 2 & 1 \\ 0 & 3 \\ 4 & 1 \end{matrix}$$

$A_{11} = 4$ | $A_{21} = +1$ | $A_{31} = -7$
$A_{12} = -4$ | $A_{22} = -6$ | $A_{32} = +2$
$A_{13} = -12$ | $A_{23} = +2$ | $A_{33} = 6$

$$\det A = 2 - 24 + 2 = -20$$

$$\text{adj}\,A = \begin{pmatrix} 4 & 1 & -7 \\ -4 & -6 & 2 \\ -12 & 2 & 6 \end{pmatrix} \qquad A^{-1} = -\frac{1}{20} \cdot \text{adj}\,A = \begin{pmatrix} -\frac{1}{5} & -\frac{1}{20} & \frac{7}{20} \\ \frac{1}{5} & \frac{3}{10} & -\frac{1}{10} \\ \frac{3}{5} & -\frac{1}{10} & -\frac{3}{10} \end{pmatrix}$$

$$\begin{pmatrix} -2 & 3 & 1 \\ 3 & 6 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

$A_{11} = 2$ $\quad A_{21} = -1$ $\quad A_{31} = 0$

$A_{12} = -1$ $\quad A_{22} = -3$ $\quad A_{32} = +7$

$A_{13} = 0$ $\quad A_{23} = +7$ $\quad A_{33} = -21$

$$\operatorname{adj} A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & -3 & 7 \\ 0 & 7 & -21 \end{pmatrix}$$

$$\left|\begin{array}{ccc|cc} -2 & 3 & 1 & -2 & 3 \\ 3 & 6 & 2 & 3 & 6 \\ 1 & 2 & 1 & 1 & 2 \end{array}\right. = -12 + 6 + \cancel{6} - \cancel{6} + 8 - 9 = -7$$

$$A^{-1} = -\frac{1}{7} \cdot \begin{pmatrix} 2 & -1 & 0 \\ -1 & -3 & 7 \\ 0 & 7 & -21 \end{pmatrix} = \begin{pmatrix} -\frac{2}{7} & +\frac{1}{7} & 0 \\ \frac{1}{7} & \frac{3}{7} & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{pmatrix}$$

$A_{11} = -6$ $\quad A_{21} = -6$ $\quad A_{31} = 3$

$A_{12} = -6$ $\quad A_{22} = 3$ $\quad A_{32} = -6$

$A_{13} = 3$ $\quad A_{23} = -6$ $\quad A_{33} = -6$

$$\operatorname{adj} A = \begin{pmatrix} -6 & -6 & 3 \\ -6 & 3 & -6 \\ 3 & -6 & -6 \end{pmatrix}$$

$$\left|\begin{array}{ccc|cc} 2 & 2 & -1 & 2 & 2 \\ 2 & -1 & 2 & 2 & -1 \\ -1 & 2 & 2 & -1 & 2 \end{array}\right. = -4 - 4 - 4 + 1 - 8 - 8 = -11 - 16 = -27$$

$$A^{-1} = -\frac{1}{27} \cdot \operatorname{adj} A = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & -\frac{1}{9} \\ \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$$

$\begin{pmatrix} 1 & 1 & 2 \\ 3 & -1 & x \\ 1 & -1 & -1 \end{pmatrix}$ x mely értékei mellett lesz inverze a mátrixnak?

$$\begin{vmatrix} 1 & 1 & 2 \\ 3 & -1 & x \\ 1 & -1 & -1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 3 & -1 \\ 1 & -1 \end{matrix} = 1 + x - 6 + 2 + x + 3 = 2x$$

$$\Downarrow$$

$$x \neq 0$$

— o —

$\begin{pmatrix} 1 & x & -12 \\ -2 & -3 & x \\ 1 & 2 & 6 \end{pmatrix}$ Mely x-ekre létezik inverze?

$$\begin{vmatrix} 1 & x & -12 \\ -2 & -3 & x \\ 1 & 2 & 6 \end{vmatrix} \begin{matrix} 1 & x \\ -2 & -3 \\ 1 & 2 \end{matrix} = -18 + x^2 + 48 - 36 - 2x + 12x = x^2 + 10x - 6 \neq 0$$

$$x_{1,2} \neq \frac{-10 \pm \sqrt{100+24}}{2} = \begin{cases} -5+\sqrt{31} \\ -5-\sqrt{31} \end{cases}$$

— o —

Mennyi a rangja?

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{pmatrix} \begin{matrix} \cdot(-2), \cdot(-3) \end{matrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -4 \\ 0 & -3 & -5 \end{pmatrix} \cdot(-3) \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -4 \\ 0 & 0 & 7 \end{pmatrix} \Rightarrow \underline{\underline{\rho(A) = 3}}$$

— o —

$$\begin{pmatrix} 2 & 1 & 2 \\ 0 & 3 & -1 \\ 4 & 1 & 1 \end{pmatrix} \cdot(-2) \sim \begin{pmatrix} 2 & 1 & 2 \\ 0 & 3 & -1 \\ 0 & -1 & -3 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 2 \\ 0 & -1 & -3 \\ 0 & 3 & -1 \end{pmatrix} (\cdot 3) \sim \begin{pmatrix} 2 & 1 & 2 \\ 0 & -1 & -3 \\ 0 & 0 & -10 \end{pmatrix} \Rightarrow \underline{\underline{\rho(A) = 3}}$$

$$\begin{pmatrix} -2 & 3 & 1 \\ 3 & 6 & 2 \\ 1 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 3 & 6 & 2 \\ -2 & 3 & 1 \end{pmatrix} \cdot(-3), (\cdot 2) \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & -1 \\ 0 & 7 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 7 & 3 \\ 0 & 0 & -1 \end{pmatrix} \Rightarrow \underline{\underline{\rho(A) = 3}}$$

$$\begin{pmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \cdot(2) \sim \begin{pmatrix} -1 & 2 & 2 \\ 0 & 3 & 6 \\ 0 & 6 & 3 \end{pmatrix} \cdot(-2) \sim \begin{pmatrix} -1 & 2 & 2 \\ 0 & 3 & 6 \\ 0 & 0 & -9 \end{pmatrix} \Rightarrow \underline{\underline{\rho(A) = 3}}$$

$\boxed{\text{7. gyakorlat (III.17) ZH!!!}}$

8. gyakorlat

utolsó előtti héten zh!

$$A=\begin{pmatrix} 2 & 5 & -1 & 4 & 3 \\ -3 & 1 & 2 & 0 & 1 \\ 4 & 1 & 6 & -1 & -1 \\ -2 & 3 & 0 & 4 & 9 \end{pmatrix} \qquad \rho(A) = \underline{\underline{4}}$$

$$\begin{pmatrix} -3 & 2 & 0 & 1 & 4 \\ -1 & 5 & 2 & 3 & 5 \\ 6 & 12 & 3 & -7 & -8 \\ -3 & 7 & 9 & 4 & 15 \end{pmatrix} \sim \begin{pmatrix} -1 & 5 & 2 & 3 & 5 \\ -3 & 2 & 0 & 1 & 4 \\ 6 & 12 & 3 & -7 & -8 \\ -3 & 7 & 9 & 4 & 15 \end{pmatrix} /(-3) \sim \begin{pmatrix} -1 & 5 & 2 & 3 & 5 \\ 0 & 13 & -6 & -8 & -11 \\ 0 & 18 & 15 & 11 & 22 \\ 0 & -8 & 3 & 5 & 0 \end{pmatrix} \begin{matrix} \cdot\frac{18}{15} \\ \\ \\ \cdot\left(\frac{8}{13}\right) \end{matrix} \sim \begin{pmatrix} -1 & 5 & 2 & 3 & 5 \\ 0 & 13 & -6 & -8 & -11 \\ 0 & 0 & 6\frac{9}{13} & -\frac{1}{13} & 6\frac{10}{13} \\ 0 & 0 & 6\frac{9}{13} & -\frac{1}{13} & 6\frac{10}{13} \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} -1 & 5 & 2 & 3 & 5 \\ 0 & 13 & -6 & -8 & -11 \\ 0 & 0 & 6\frac{9}{13} & -\frac{1}{13} & 6\frac{10}{13} \end{pmatrix}$$

Lineáris egyenletrendszerek megoldása

1;

$$\begin{aligned} x_1 - x_2 \qquad &= 0 \\ x_2 + x_3 &= 3 \\ x_1 + 2x_2 - x_3 &= 1 \\ x_1 + \qquad + 2x_3 &= 5 \end{aligned}$$

4 sor és 3 oszlop.

A Cramer-szabály akkor alkalmazható, ha a mátrix négyzetes és a determinánsa nem 0.

Gauss elimináció

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 1 & 0 & 2 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 3 & -1 & 1 \\ 0 & 1 & 2 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ \cancel{0} & \cancel{0} & \cancel{-4} & \cancel{-8} \\ 0 & 0 & 1 & 2 \end{pmatrix} \sim \begin{array}{c} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right) \\ \begin{array}{ccc|c} x_1 & x_2 & x_3 & c \end{array} \end{array}$$

A 3. sor -4x-ese a 4. sornak ⇒ elhagyható

$x_3 = 2$

$x_2 = 1$

$x_1 = 1.$

Egy megoldása van az er-nek, így HATÁROZOTT.

2.,
$$\begin{cases} \overset{\alpha_{11}}{2x_1} - x_2 - 2x_3 = \overset{\beta_1}{5} \\ 4x_1 + x_2 + 2x_3 = 1 \\ 8x_1 - x_2 + x_3 = 5 \end{cases}$$

Alapmátrix: Az ismeretlenek együtthatóiból képzett mátrix.

Cramer-szab. szerint:

$$\begin{pmatrix} 2 & -1 & -2 \\ 4 & 1 & 2 \\ 8 & -1 & 1 \end{pmatrix}$$

$|A| \neq 0$

A négyzetes

$\Big\} \Rightarrow$ er. megoldható és határozott, $x_k = \frac{D_k}{A}$,

$$D_k = \begin{array}{c} \phantom{\alpha_{11} \cdots} k \phantom{\cdots \alpha_{1n}} \\ \begin{vmatrix} \alpha_{11} & \cdots & \beta_1 & \cdots & \alpha_{1n} \\ \vdots & & & & \\ \alpha_{n1} & \cdots & \beta_n & \cdots & \alpha_{nn} \end{vmatrix} \end{array}$$

$$|A| = \begin{vmatrix} 2 & -1 & -2 \\ 4 & 1 & 2 \\ 8 & -1 & 1 \end{vmatrix} = \underline{\underline{18 \neq 0}}$$

$$D_1 = \begin{vmatrix} 5 & -1 & -2 \\ 1 & 1 & 2 \\ 5 & -1 & 1 \end{vmatrix} = 18$$

$$D_2 = \begin{vmatrix} 2 & 5 & -2 \\ 4 & 1 & 2 \\ 8 & 5 & 1 \end{vmatrix} = 18 \qquad x_2 = \frac{D_2}{|A|} = \underline{\underline{1}}$$

$$D_3 = \begin{vmatrix} 2 & -1 & 5 \\ 4 & 1 & 1 \\ 8 & -1 & 5 \end{vmatrix} = -36 \qquad x_3 = \frac{D_3}{|A|} = \underline{\underline{-2}}$$

$$x_1 = \frac{D_1}{|A|} = \frac{18}{18} = \underline{\underline{1}}$$

3; $\begin{cases} x_1 + 2x_2 + 3x_3 = 5 \\ 2x_1 - x_2 + x_3 = 1 \end{cases}$

kibővített mátrix:

$$\begin{pmatrix} 1 & 2 & 3 & 5 \\ 2 & -1 & 1 & 1 \end{pmatrix} \sim \begin{array}{c} \begin{pmatrix} 1 & 2 & 3 & 5 \\ 0 & -5 & -5 & -9 \end{pmatrix} \\ \begin{array}{cccc} x_1 & x_2 & x_3 & c \end{array} \end{array}$$

$$-5x_2 - 5x_3 = -9$$

jelöljük az x_3-t „t"-vel.

$$-5x_2 - 5t = -9$$

$$-5x_2 = -9 + 5t$$

$$x_2 = \frac{9}{5} - t$$

$$x_1 = 5 - 2x_2 - 3x_3$$

$$x_1 = 5 - 2\left(\frac{9}{5} - t\right) - 3t$$

$$x_1 = 5 - \frac{18}{5} + 2t - 3t$$

$$x_1 = \frac{7}{5} - t$$

határozatlan

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t \underbrace{\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}_{\text{együtthatók}} + \underbrace{\begin{pmatrix} \frac{7}{5} \\ \frac{9}{5} \\ 0 \end{pmatrix}}_{\text{konstansok}}$$

4.,
$$\begin{aligned} x_1 + 3x_2 - 3x_3 &= 0 \\ x_2 - 2x_3 &= 0 \\ -x_1 - 3x_3 &= 0 \\ 2x_1 + x_2 + 4x_3 &= 0 \\ x_1 + x_2 + x_3 &= 0 \end{aligned}$$

kibőv. mátrix:

$$\begin{pmatrix} 1 & 3 & -3 & 0 \\ 0 & 1 & -2 & 0 \\ -1 & 0 & -3 & 0 \\ 2 & 1 & 4 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & -3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 3 & -6 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & -2 & 4 & 0 \end{pmatrix} \sim \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \begin{pmatrix} 1 & 3 & -3 & 0 \\ 0 & 1 & -2 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix} \end{array}$$

$$x_2 - 2x_3 = 0$$

$$x_3 := t$$

$$x_2 = 2x_3 = 2t$$

$x_1 = -3x_3 = x_1 = -3t \Rightarrow$ 3. sorból!

határozatlan

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$$

5,

$$\begin{cases} x_1 + 2x_2 + x_3 - x_4 = 3 \\ -x_1 + x_2 - 2x_3 + 2x_4 = 0 \end{cases}$$

$$\begin{pmatrix} 1 & 2 & 1 & -1 & 3 \\ -1 & 1 & -2 & 2 & 0 \end{pmatrix} \sim \begin{array}{c} \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 3 \\ 0 & 3 & -1 & 1 & 3 \end{array}\right) \\ \begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & c \end{array} \end{array}$$

$$3x_2 - x_3 + x_4 = 3$$

$$x_4 := t$$
$$x_3 := v$$

$$x_2 = (3 + x_3 - x_4)/3 = 1 + \frac{v}{3} - \frac{t}{3}$$

$$x_1 = 3 - 2x_2 - x_3 + x_4 = 3 - 2 - \frac{2v}{3} + \frac{2t}{3} - v + t = 1 - \frac{5v}{3} + \frac{5t}{3}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t\begin{pmatrix} \frac{5}{3} \\ \frac{-1}{3} \\ 0 \\ 1 \end{pmatrix} + v\begin{pmatrix} \frac{5}{3} \\ \frac{1}{3} \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

6.,

$$\begin{cases} 8x_1 + 2x_2 + 9x_3 + 5x_4 = 0 \\ 4x_1 + x_2 + 3x_3 + x_4 = 0 \\ 8x_1 + 2x_2 + 5x_3 + x_4 = 0 \end{cases}$$

$$\begin{pmatrix} 8 & 2 & 9 & 5 & 0 \\ 4 & 1 & 3 & 1 & 0 \\ 8 & 2 & 5 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 4 & 1 & 3 & 1 & 0 \\ 8 & 2 & 9 & 5 & 0 \\ 8 & 2 & 5 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 4 & 1 & 3 & 1 & 0 \\ 0 & 0 & 3 & 3 & 0 \\ \cancel{0} & \cancel{0} & \cancel{1} & \cancel{1} & \cancel{0} \end{pmatrix} \sim \begin{array}{c} \left(\begin{array}{cccc|c} 4 & 1 & 3 & 1 & 0 \\ 0 & 0 & 3 & 3 & 0 \end{array}\right) \\ \begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & c \end{array} \end{array}$$

$$3x_3 + 3x_4 = 0$$

$$x_4 := t$$

$$3x_3 = -3t$$
$$x_3 = -t$$

$$8x_1 + 2x_2 + 5x_3 + x_4 = 0 = 8x_1 + 2x_2 - 5t + t = \boxed{8x_1 + 2x_2 - 4t = 0}$$

$$8x_1 + 2x_2 + 9x_3 + 5x_4 = 0 = 8x_1 + 2x_2 - 9t + 5t = \boxed{8x_1 + 2x_2 - 4t = 0}$$

$$x_2 := v$$

$$8x_1 + 2v - 4t = 0$$
$$8x_1 = 4t - 2v$$
$$x_1 = \frac{1}{2}t - \frac{1}{4}v$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t\begin{pmatrix} \frac{1}{2} \\ 0 \\ -1 \\ 1 \end{pmatrix} + v\begin{pmatrix} -\frac{1}{4} \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

határozatlan

7;

$$\begin{cases} 2x_1 + x_2 - 4x_3 = 0 \\ 3x_1 + 5x_2 - 7x_3 = 0 \\ 4x_1 - 5x_3 - 6x_3 = 0 \end{cases}$$

$$\begin{vmatrix} 2 & 1 & -4 \\ 3 & 5 & -7 \\ 4 & -5 & -6 \end{vmatrix} = -60 + 60 - 28 + \underbrace{80 - 70}_{10} + 18 = \emptyset$$

$$\begin{matrix} 2 & 1 & -4 \\ 3 & 5 & -7 \end{matrix}$$

Gauss-elimináció!

$$\begin{pmatrix} 2 & 1 & -4 & 0 \\ 3 & 5 & -7 & 0 \\ 4 & -5 & -6 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -4 & 0 \\ 5 & 3 & -7 & 0 \\ -5 & 4 & -6 & 0 \end{pmatrix} \sim \begin{array}{cccc} x_1 & x_2 & x_3 & c \\ \end{array} \begin{pmatrix} 1 & 2 & -4 & 0 \\ 0 & -7 & 13 & 0 \\ \cancel{0} & \cancel{14} & \cancel{-26} & \cancel{0} \end{pmatrix}$$

$$-7x_2 - 13x_3 = 0$$

$$-7x_2 = 13x_3$$

$$x_3 := t \qquad x_2 = \frac{-13t}{7}$$

$$2x_1 + x_2 - 4x_3 = 0$$

$$2x_1 = 4x_3 - x_2$$

$$x_1 = 2x_3 - \frac{x_2}{2} = -\frac{15}{14}t$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t \begin{pmatrix} -\frac{15}{14} \\ \frac{13}{7} \\ 1 \end{pmatrix}$$

Hf: $$\begin{cases} 3x_1 - 2x_2 + x_3 + 2x_4 = 1 \\ x_1 + x_2 - x_3 - x_4 = -2 \\ 2x_1 - x_2 + 3x_3 = 4 \end{cases}$$

$$\begin{cases} 2x_1 + 7x_2 + 3x_3 + x_4 = 6 \\ 3x_1 + 5x_2 + 2x_3 + 2x_4 = 4 \\ 9x_1 + 4x_2 + x_3 + 7x_4 = 2 \end{cases}$$

$$\begin{cases} 2x_1 + 5x_2 - 8x_3 = 8 \\ 4x_1 + 3x_2 - 9x_3 = 9 \\ 2x_1 + 3x_2 - 5x_3 = 7 \\ x_1 + 8x_2 - 7x_3 = 12 \end{cases}$$

$$\begin{cases} 3x_1 + 2x_2 + 2x_3 + 2x_4 = 2 \\ 2x_1 + 3x_2 + 2x_3 + 5x_4 = 3 \\ 9x_1 + x_2 + 4x_3 - 5x_4 = 1 \\ 2x_1 + 2x_2 + 3x_3 + 4x_4 = 5 \\ 7x_1 + x_2 + 6x_3 - x_4 = 7 \end{cases}$$

Hf.: $\begin{aligned} 3x_1 - 2x_2 + x_3 + 2x_4 &= 1 \\ x_1 + x_2 - x_3 - x_4 &= -2 \\ 2x_1 - x_2 + 3x_3 &= 4 \end{aligned}$

$$\begin{pmatrix} 3 & -2 & 1 & 2 & 1 \\ 1 & 1 & -1 & -1 & -2 \\ 2 & -1 & 3 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 & -1 & -2 \\ 2 & -1 & 3 & 0 & 4 \\ 3 & -2 & 1 & 2 & 1 \end{pmatrix} \overset{(\cdot 2)}{\sim} \begin{pmatrix} -1 & 1 & -1 & 1 & -2 \\ 0 & -1 & 3 & 2 & 4 \\ 2 & -2 & 1 & 3 & 1 \end{pmatrix} \sim$$

$$\sim \left(\begin{array}{cccc|c} -1 & 1 & -1 & 1 & -2 \\ 0 & -1 & 3 & 2 & 4 \\ 0 & 0 & -1 & 5 & 3 \end{array}\right)$$
$$\begin{array}{ccccc} & & x_3 & x_4 & c \end{array}$$

$$-x_3 + 5x_4 = 3$$

$$x_3 = 5x_4 - 3 \qquad x_4 := t \quad \rightarrow x_3 = 5t - 3$$

$$x_1 + x_2 - x_3 - x_4 = -2$$

$$x_1 + x_2 - 5t - 3 - t = -2$$

$$x_1 + x_2 - 6t = 1 \qquad x_2 := v$$
$$x_1 = 1 + 6t - v$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t \begin{pmatrix} 6 \\ 0 \\ 5 \\ 1 \end{pmatrix} + v \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -3 \\ 0 \end{pmatrix}$$

hatlan

$\begin{aligned} 2x_1 + 7x_2 + 3x_3 + x_4 &= 6 \\ 3x_1 + 5x_2 + 2x_3 + 2x_4 &= 4 \\ 9x_1 + 4x_2 + x_3 + 7x_4 &= 2 \end{aligned}$

$$\begin{pmatrix} 2 & 7 & 3 & 1 & 6 \\ 3 & 5 & 2 & 2 & 4 \\ 9 & 4 & 1 & 7 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 7 & 4 & 9 \\ 2 & 4 & 2 & 5 & 3 \\ 3 & 6 & 1 & 7 & 2 \end{pmatrix} \overset{(-2)\ (-3)}{\sim} \left(\begin{array}{cccc|c} 1 & 2 & 7 & 4 & 9 \\ 0 & 0 & -12 & -3 & -15 \\ \cancel{0} & \cancel{0} & \cancel{-20} & \cancel{-5} & \cancel{-25} \end{array}\right)$$

$$-12x_3 - 3x_4 = -15 \qquad /:-3$$

$$4x_3 + x_4 = 5$$
$$x_4 = 5 - 4x_3 \qquad x_3 := t$$
$$x_4 = 5 - 4t$$

$$2x_1 + 7x_2 + 3t + 5 - 4t = 6$$
$$2x_1 + 7x_2 + 5 - t = 6 \qquad x_2 := v$$
$$2x_1 + 7v + 5 - t = 6$$
$$2x_1 = 1 - t - 7v$$
$$x_1 = 1/2 - t/2 - 7/2\, v$$