1.

1. gyakorlat

$(G;*)$ alg. struktúra félcsoport, ha • $*$ műv. asszociatív

— " — csoport, ha • $*$ műv asszociatív.
• $\exists$ G-ben neutrális elem
• $\exists$ minden G-beli elemnek inverze

$(G;*)$ alg. struktúra komm. csop, ha • csoport
• ha $*$ műv. kommutatív

$(R;+;\cdot)$ alg. struktúra gyűrű, ha • $(R,+)$ komm. csoport
• $(R,\cdot)$ félcsoport
• disztributivitás

$(T;+;\cdot)$ test, ha • $(T;+)$ komm. csoport
• $(T,\cdot)$ komm.. csoport
• disztributivitás

Def.: Az $\{1,2\ldots n\}$ elem önmagára való kölcsönösen egyértelmű leképezését PERMUTÁCIÓnak nevezzük.

jel.: $\begin{pmatrix} 1,2,3,4\ldots n \\ i_1,i_2,\ldots i_n \end{pmatrix} = (i_1, i_2, \ldots i_n) = \pi$

pl: $\begin{pmatrix} 1, 2, 3, 4, 5 \\ 3, 2, 4, 5, 1 \end{pmatrix}$

Def. Az $(i_1, i_2, \ldots i_n)$ permutációban az i_k elem INVERZIÓban áll i_l-vel, ha $k<l$, de $i_k > i_l$

pl.: $\begin{pmatrix} 1, 2, 3, 4, 5 \\ 3, 2, 4, 5, 1 \end{pmatrix}$
i_1, i_2, i_3, i_4, i_5

$i_1 < i_2$
$3 > 2$

inverzió: $\left.\begin{matrix} i_1\, i_2 \\ i_1\, i_5 \\ i_2\, i_5 \\ i_3\, i_5 \\ i_4\, i_5 \end{matrix}\right\}$ 5 db

Az inverzió paritása adja meg a permutáció paritását.
(Ha az páratlan (lsd.: 5) $\Rightarrow$ a perm. is páratlan)

Def.: Egy permutációban az inverziók paritása megadja a permutáció paritását.

Def.: PERMUTÁCIÓK SZORZÁSA:

$$\begin{pmatrix} 1 & 2 & 3 & \dots & n \\ i_1 & i_2 & i_3 & \dots & i_n \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ j_1 & j_2 & j_3 & \dots & j_n \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ j_{i_1} & j_{i_2} & j_{i_3} & \dots & j_{i_n} \end{pmatrix}$$

pl.: $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$

Állítás: A permutációk a permutáció szorzás műveletére nézve csoportot alkotnak.

egységelem: $\begin{pmatrix} 1 & 2 & 3 \dots & n \\ 1 & 2 & 3 \dots & n \end{pmatrix}$

inverzelem: $\begin{pmatrix} 1 & 2 & 3 & \dots & n \\ i_1 & i_2 & i_3 & \dots & i_n \end{pmatrix}^{-1} = \begin{pmatrix} i_1, & i_2, & i_3 & \dots & i_n \\ 1 & 2 & 3 & \dots & n \end{pmatrix}$

BIZ: Hf. (csoport)

Ha csoport $\Rightarrow \exists$ egységelem

$\begin{pmatrix} 1 & 2 & 3 \dots & n \\ 1 & 2 & 3 \dots & n \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \dots & n \\ 2 & 3 & 1 \dots & n \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \dots & n \\ 2 & 3 & 1 \dots & n \end{pmatrix}$ ✓

$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$ ✓

Tehát csoport!

11.10.

2. gyakorlat

Mátrixok:

$A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 4 & 0 & 2 & 1 \\ 2 & -5 & 1 & 2 \end{pmatrix}_{3\times4}$ $\qquad B = \begin{pmatrix} 3 & -4 & 1 & 2 \\ 1 & 5 & 0 & 3 \\ 2 & -2 & 3 & -1 \end{pmatrix}_{3\times4}$

$A+B=?$ $\qquad A-B=?$ $\qquad 3A=?$ $\qquad -B=?$

$A+B = \begin{pmatrix} 4 & -2 & 0 & 2 \\ 5 & 5 & 2 & 4 \\ 4 & -7 & 4 & 1 \end{pmatrix}_{3\times4}$

$A-B = \begin{pmatrix} -2 & 6 & -2 & -2 \\ 3 & -5 & 2 & -2 \\ 0 & -3 & -2 & 3 \end{pmatrix}_{3\times4}$

3.

$$3A = \begin{pmatrix} 3 & 6 & -3 & 0 \\ 12 & 0 & 6 & 3 \\ 6 & -15 & 3 & 6 \end{pmatrix}_{3\times 4}$$

$$-B = \begin{pmatrix} -3 & 4 & -1 & -2 \\ -1 & -5 & 0 & -3 \\ -2 & 2 & -3 & 1 \end{pmatrix}_{3\times 4}$$

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}_{4\times 2} \qquad B = \begin{pmatrix} 4 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}_{2\times 3} \qquad A \times B = ?$$

Falk-módszer:

$$\begin{array}{r|l} & B = \begin{pmatrix} 4 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}_{2\times 3} \\ \hline A = \begin{pmatrix} 1 & 3 \\ 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}_{4\times 2} & \begin{pmatrix} 4 & -3 & -1 \\ 8 & 0 & -2 \\ -4 & -1 & 1 \\ 0 & -2 & 0 \end{pmatrix}_{4\times 3} = A \times B \end{array}$$

$$A = \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{pmatrix} \qquad \begin{array}{l} A \times B = ? \\ B \times A = ? \end{array}$$

$$\begin{array}{r|l} & \begin{pmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{pmatrix}_{3\times 3} = B \\ \hline A = \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix}_{3\times 3} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{3\times 3} = A \times B \end{array}$$

A mátrixgyűrű nem zérusosztómentes!

$A \times B = AB$

↳ zérusmátrix, de A és B nem zérusmátrix

$$\begin{array}{r|l} & \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix}_{3\times 3} = A \\ \hline B = \begin{pmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{pmatrix}_{3\times 3} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{3\times 3} = A \times B \end{array}$$

$$A=\begin{pmatrix}3 & 1 & -1\\ -1 & 0 & 2\\ 1 & 2 & 2\end{pmatrix}_{3\times3} \qquad B=\begin{pmatrix}2 & -1\\ 3 & 4\\ 1 & 0\end{pmatrix}_{3\times2}$$

$A\times B = ?$

$$\begin{array}{r|l} & \begin{pmatrix}2 & -1\\ 3 & 4\\ 1 & 0\end{pmatrix}_{3\times2}=B \\ \hline A=\begin{pmatrix}3 & 1 & -1\\ -1 & 0 & 2\\ 1 & 2 & 2\end{pmatrix}_{3\times3} & \begin{pmatrix}8 & 1\\ 0 & 1\\ 10 & 7\end{pmatrix}_{3\times2}=A\times B\end{array}$$

— o —

$$A=\begin{pmatrix}3 & -2 & -1\\ -2 & 4 & 2\\ 3 & 1 & -2\end{pmatrix}_{3\times3} \qquad B=\begin{pmatrix}2 & 3 & -4\\ 2 & -1 & 2\\ -1 & 1 & 3\end{pmatrix}_{3\times3}$$

Igazoljuk, hogy $(A+B)^2 = A^2+AB+BA+B^2$.

$$A+B=\begin{pmatrix}5 & +1 & -5\\ 0 & 3 & 4\\ 2 & 2 & 1\end{pmatrix}$$

$$\begin{array}{r|l} & \begin{pmatrix}5 & 1 & -5\\ 0 & 3 & 4\\ 2 & 2 & 1\end{pmatrix}_{3\times3}=A+B \\ \hline A+B=\begin{pmatrix}5 & 1 & -5\\ 0 & 3 & 4\\ 2 & 2 & 1\end{pmatrix}_{3\times3} & \begin{pmatrix}15 & -2 & -26\\ 8 & 17 & 16\\ 12 & 10 & -1\end{pmatrix}_{3\times3}=(A+B)^2\end{array}$$

$A^2 = ?$

$$\begin{array}{r|l} & \begin{pmatrix}3 & -2 & -1\\ -2 & 4 & 2\\ 3 & 1 & -2\end{pmatrix}=A \\ \hline A=\begin{pmatrix}3 & -2 & -1\\ -2 & 4 & 2\\ 3 & 1 & -2\end{pmatrix} & \begin{pmatrix}10 & -15 & -5\\ -8 & 22 & 6\\ 1 & -4 & 3\end{pmatrix}_{3\times3}=A^2\end{array} \qquad \begin{array}{r|l} & \begin{pmatrix}2 & 3 & -4\\ 2 & -1 & 2\\ -1 & 1 & 3\end{pmatrix}=B \\ \hline B=\begin{pmatrix}2 & 3 & -4\\ 2 & -1 & 2\\ -1 & 1 & 3\end{pmatrix} & \begin{pmatrix}14 & -1 & -14\\ 0 & 9 & -4\\ -3 & -1 & 15\end{pmatrix}_{3\times3}=B^2\end{array}$$

$$\begin{array}{r|l} & \begin{pmatrix}2 & 3 & -4\\ 2 & -1 & 2\\ -1 & 1 & 3\end{pmatrix}=B \\ \hline A=\begin{pmatrix}3 & -2 & -1\\ -2 & 4 & 2\\ 3 & 1 & -2\end{pmatrix} & \begin{pmatrix}3 & 10 & -19\\ 2 & -8 & 22\\ 10 & 6 & -16\end{pmatrix}_{3\times3}=A\times B\end{array} \qquad \begin{array}{r|l} & \begin{pmatrix}3 & -2 & -1\\ -2 & 4 & 2\\ 3 & 1 & -2\end{pmatrix}=A \\ \hline B=\begin{pmatrix}2 & 3 & -4\\ 2 & -1 & 2\\ -1 & 1 & 3\end{pmatrix} & \begin{pmatrix}-12 & +4 & 12\\ 14 & -6 & -8\\ 4 & 9 & -3\end{pmatrix}=B\times A\end{array}$$

$$A^2+AB+BA+B^2=\begin{pmatrix}10 & -15 & -5\\ -8 & 22 & 6\\ 1 & -4 & 3\end{pmatrix}+\begin{pmatrix}3 & 10 & -19\\ 2 & -8 & 22\\ 10 & 6 & -16\end{pmatrix}+\begin{pmatrix}-12 & 4 & 12\\ 14 & -6 & -8\\ 4 & 9 & -3\end{pmatrix}+\begin{pmatrix}14 & -1 & -14\\ 0 & 9 & -4\\ -3 & -1 & 15\end{pmatrix}=\begin{pmatrix}15 & -2 & -26\\ 8 & 17 & 16\\ 12 & 10 & -1\end{pmatrix}$$

$$\Downarrow$$

$$(A+B)^2$$

5. $A = \begin{pmatrix} 1 & -3 & 2 \\ 2 & -1 & 4 \end{pmatrix}_{2\times3}$ $\quad B = \begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix}_{3\times1}$ $\quad C = \begin{pmatrix} -1 & 1 & 2 & -1 \end{pmatrix}_{1\times4}$

Igazoljuk, hogy az asszociativitás teljesül!

$(A\cdot B)\cdot C = A\cdot(B\cdot C)$

az egyik oszlopai = a másik soraival.

$$\begin{array}{c|c} & \begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix}_{3\times1} = B \\ \hline A = \begin{pmatrix} 1 & -3 & 2 \\ 2 & -1 & 4 \end{pmatrix}_{2\times3} & \begin{pmatrix} -12 \\ -14 \end{pmatrix}_{2\times1} = A\times B \end{array}$$

$$\begin{array}{c|c} & \begin{pmatrix} -1 & 1 & 2 & -1 \end{pmatrix} = C \\ \hline A\times B = \begin{pmatrix} -12 \\ -14 \end{pmatrix} & \begin{pmatrix} +12 & -12 & -24 & 12 \\ 14 & -14 & -28 & 14 \end{pmatrix}_{2\times4} = (AB)C \end{array}$$

$$\begin{array}{c|c} & C = \begin{pmatrix} -1 & 1 & 2 & -1 \end{pmatrix}_{1\times4} \\ \hline B = \begin{pmatrix} -4 \\ 2 \\ -1 \end{pmatrix}_{3\times1} & \begin{pmatrix} 4 & -4 & -8 & 4 \\ -2 & 2 & 4 & -2 \\ 1 & -1 & -2 & 1 \end{pmatrix}_{3\times4} = B\cdot C \end{array}$$

$$\begin{array}{c|c} & \begin{pmatrix} 1 & -3 & 2 \\ 2 & -1 & 4 \end{pmatrix}_{2\times3} = A \\ \hline B\times C \begin{pmatrix} 4 & -4 & -8 & 4 \\ -2 & 2 & 4 & -2 \\ 1 & -1 & -2 & 1 \end{pmatrix}_{3\times4} & \end{array}$$

$\Downarrow$

fordítva nem szorozható

$$\begin{array}{c|c} & \begin{pmatrix} 4 & -4 & -8 & 4 \\ -2 & 2 & 4 & -2 \\ 1 & -1 & -2 & 1 \end{pmatrix} = B\times C \\ \hline A = \begin{pmatrix} 1 & -3 & 2 \\ 2 & -1 & 4 \end{pmatrix}_{2\times3} & \begin{pmatrix} 12 & -12 & -24 & 12 \\ 14 & -14 & -28 & 14 \end{pmatrix} A(B\times C) \end{array}$$

$\Downarrow$

Tehát:

$A(BC) = (AB)C$

$A = \begin{pmatrix} 4 & 3 & 6 \\ 5 & 0 & 1 \end{pmatrix}_{2\times3}$ $\quad B = \begin{pmatrix} 4 & 3 \\ 5 & 0 \\ 6 & 1 \end{pmatrix}_{3\times2}$

Melyik igaz?

a, $A = B$ H

b, $AB = E$ → egységmátrix H

c, A^T = (transzponált) B

d, $A = B^T$

Transzponált: a főátlóra tükrözzük az elemeket.

pl.:
$$A = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ & & \vdots & \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{pmatrix}_{m \times n}$$

pl.:
$$A^T = \begin{pmatrix} \alpha_{11} & \alpha_{21} & \cdots & \alpha_{m1} \\ \alpha_{12} & \alpha_{22} & \cdots & \alpha_{m2} \\ & & \vdots & \\ \alpha_{1n} & \alpha_{2n} & \cdots & \alpha_{mn} \end{pmatrix}_{m \times n}$$

pl.: $A = \begin{pmatrix} 1 & 2 & 3 \\ 7 & 5 & 6 \\ 10 & 4 & 9 \end{pmatrix} \rightarrow A^T = \begin{pmatrix} 1 & 7 & 10 \\ 2 & 5 & 4 \\ 3 & 6 & 9 \end{pmatrix}$

pl.: $A = \begin{pmatrix} 1 & 4 & 5 \\ 2 & 7 & 6 \end{pmatrix}_{2 \times 3} \rightarrow A^T = \begin{pmatrix} 1 & 2 \\ 4 & 7 \\ 5 & 6 \end{pmatrix}$

$A = \begin{pmatrix} 4 & 3 & 6 \\ 5 & 0 & 1 \end{pmatrix} \rightarrow A^T = \begin{pmatrix} 4 & 5 \\ 3 & 0 \\ 6 & 1 \end{pmatrix}$

$B = \begin{pmatrix} 4 & 3 \\ 5 & 0 \\ 6 & 1 \end{pmatrix}_{2 \times 3} \qquad B^T = \begin{pmatrix} 4 & 5 & 6 \\ 3 & 0 & 1 \end{pmatrix}$

$A^T = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 1 & 0 \end{pmatrix}_{2 \times 3} \qquad B = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}_{1 \times 3}$

? $AB = \begin{pmatrix} 7 & 4 \end{pmatrix}_{1 \times 2}$

$A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \\ 4 & 0 \end{pmatrix}_{3 \times 2}$

Nem tudjuk az A és B mátrixokat összeszorozni!!!

Hf.:

$A = \begin{pmatrix} 4 & 2 & -1 & 2 \\ 3 & -7 & 1 & -8 \\ 2 & 4 & -3 & 1 \end{pmatrix}_{3 \times 4} \qquad B = \begin{pmatrix} 2 & 3 \\ -3 & 0 \\ 1 & 5 \\ 3 & 1 \end{pmatrix}_{4 \times 2} \qquad A \times B = ?$

$$\begin{array}{c|c} & \begin{pmatrix} -3 & 0 \\ 1 & 5 \\ 3 & 1 \end{pmatrix}_{4\times 2} = B \\ \hline A = \begin{pmatrix} 4 & 2 & -1 & 2 \\ 3 & -7 & 1 & -8 \\ 2 & 4 & -3 & 1 \end{pmatrix}_{3\times 4} & \begin{pmatrix} 7 & 9 \\ 4 & 6 \\ -8 & -8 \end{pmatrix}_{3\times 2} \end{array}$$

3. gyakorlat

11.17.

$$A = \begin{pmatrix} 3 & -2 & -1 \\ -2 & 4 & 2 \\ 3 & 1 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 3 & -4 \\ 2 & -1 & 2 \\ -1 & 1 & -3 \end{pmatrix}$$

Igazoljuk: $A^2 - B^2 \neq (A-B)(A+B)$

$(A+B)(A-B) = A^2 + AB - BA - B^2$

$$A^2 - B^2 = \begin{pmatrix} -4 & -14 & -15 \\ -8 & 13 & 22 \\ -2 & 3 & -12 \end{pmatrix}$$

$$A - B = \begin{pmatrix} 1 & -5 & 3 \\ -4 & 5 & 0 \\ 4 & 0 & 1 \end{pmatrix} \qquad A+B = \begin{pmatrix} 5 & 1 & -5 \\ 0 & 3 & 4 \\ 2 & 2 & -5 \end{pmatrix}$$

$$\begin{array}{c|c} & \begin{pmatrix} 5 & 1 & -5 \\ 0 & 3 & 4 \\ 2 & 2 & -5 \end{pmatrix} = (A+B) \\ \hline (A-B) = \begin{pmatrix} 1 & -5 & 3 \\ -4 & 5 & 0 \\ 4 & 0 & 1 \end{pmatrix} & \begin{pmatrix} 11 & -8 & -40 \\ -20 & 11 & 40 \\ 22 & 6 & -25 \end{pmatrix} = (A-B)(A+B) \end{array}$$

~~~~~ o ~~~~~

$$A = \begin{pmatrix} 1 & 1-i \\ 2i & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 4i & 1+2i \\ 4 & 3-i \end{pmatrix} \qquad C = \begin{pmatrix} -2i & 1 \\ 1 & 1+i \end{pmatrix}$$

Számoljuk ki: $i\cdot A - 2B + 3C = ?$

$$iA = \begin{pmatrix} i & i - \overset{\nearrow i+1}{i^2} \\ \underset{\searrow -2}{2i^2} & 3i \end{pmatrix} \qquad 2B = \begin{pmatrix} 8i & 2+4i \\ 8 & 6-2i \end{pmatrix} \qquad 3C = \begin{pmatrix} -6i & 3 \\ 3 & 3+3i \end{pmatrix}$$

$$i\cdot A - 2B + 3C = \begin{pmatrix} -13i & 2-3i \\ -7 & 8i-3 \end{pmatrix}$$
~~~~~

$$A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \qquad A^2 = ?$$

$$\begin{array}{c|c} & \begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = A \\ \hline A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} & \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha + \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix} \end{array} = \begin{pmatrix} 1 & \sin 2\alpha \\ \sin 2\alpha & 1 \end{pmatrix} = A^2$$

~~~~~ o ~~~~~

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & c \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix} = B \qquad c = ?$$

AB legyen szimmetrikus

$$\begin{array}{c|c} & \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix} = B \\ \hline A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & c \end{pmatrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c & 2c & 3c \end{matrix} \end{array} \rightarrow \underline{\underline{c = 0}}$$

$$\begin{matrix} 1 & 2 & 4 \\ -2 & 3 & 5 \\ -4 & -5 & 6 \end{matrix}$$

Antiszimmetrikus az átlóra szimmetrikus elemek csak az előjelekben térnek el.

~~~~~ o ~~~~~

$$f(x) = 3x^2 - 2x + 5x^0 \qquad f(x) = ?$$

$$x = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -4 & 1 \\ 3 & -5 & 2 \end{pmatrix}$$

$$\begin{array}{c|c} & \begin{pmatrix} 1 & -2 & 3 \\ 2 & -4 & 1 \\ 3 & -5 & 2 \end{pmatrix} = x \\ \hline x = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -4 & 1 \\ 3 & -5 & 2 \end{pmatrix} & \begin{pmatrix} 6 & -9 & 7 \\ -3 & 7 & 4 \\ -1 & 4 & 8 \end{pmatrix} = x^2 \end{array} \qquad 3x^2 = \begin{pmatrix} 18 & -27 & 21 \\ -9 & 21 & 12 \\ -3 & 12 & 24 \end{pmatrix}$$

$$-2x = \begin{pmatrix} -2 & 4 & -6 \\ -4 & 8 & -2 \\ -6 & 10 & -4 \end{pmatrix} \qquad x^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad 5x^0 = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

9

$$f(x) = 3x^2 - 2x + 5x^0$$

$$f\,x = \begin{pmatrix} 21 & -23 & 15 \\ -13 & 34 & 10 \\ -9 & 22 & 25 \end{pmatrix}$$

$$C = \begin{pmatrix} \pi & 0 & 0 \\ 0 & \pi^2 & 0 \\ 0 & 0 & \pi^3 \end{pmatrix} \qquad C^{1000} = ?$$

$$\begin{pmatrix} \pi & 0 & 0 \\ 0 & \pi^2 & 0 \\ 0 & 0 & \pi^3 \end{pmatrix} \cdot \begin{pmatrix} \pi & 0 & 0 \\ 0 & \pi^2 & 0 \\ 0 & 0 & \pi^3 \end{pmatrix} = \begin{pmatrix} \pi^2 & 0 & 0 \\ 0 & \pi^4 & 0 \\ 0 & 0 & \pi^6 \end{pmatrix} = C^2$$

$$\begin{pmatrix} \pi^2 & 0 & 0 \\ 0 & \pi^4 & 0 \\ 0 & 0 & \pi^6 \end{pmatrix} \cdot \begin{pmatrix} \pi & 0 & 0 \\ 0 & \pi^2 & 0 \\ 0 & 0 & \pi^3 \end{pmatrix} = \begin{pmatrix} \pi^3 & 0 & 0 \\ 0 & \pi^6 & 0 \\ 0 & 0 & \pi^9 \end{pmatrix} = C^3$$

$$\begin{pmatrix} \pi^3 & 0 & 0 \\ 0 & \pi^6 & 0 \\ 0 & 0 & \pi^9 \end{pmatrix} \cdot \begin{pmatrix} \pi & 0 & 0 \\ 0 & \pi^2 & 0 \\ 0 & 0 & \pi^3 \end{pmatrix} = \begin{pmatrix} \pi^4 & 0 & 0 \\ 0 & \pi^8 & 0 \\ 0 & 0 & \pi^{12} \end{pmatrix} = C^4$$

C^2: a Eikrő 2x annyi
C^3: —"— 3x —"—
C^4: —"— 4x —"—
C^{1000}: —"— 1000x —"—

$$C^{1000} = \begin{pmatrix} \pi^{1000} & 0 & 0 \\ 0 & \pi^{2000} & 0 \\ 0 & 0 & \pi^{3000} \end{pmatrix}$$

Biz be! $\qquad n \in \mathbb{N}$

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}^n = \begin{pmatrix} \cos(n\alpha) & -\sin(n\alpha) \\ \sin(n\alpha) & \cos(n\alpha) \end{pmatrix}$$

1.) $n = 2$ ✓

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos^2\alpha - \sin^2\alpha & -\sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cdot\cos\alpha + \cos\alpha\sin\alpha & -\sin^2\alpha + \cos^2\alpha \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \overset{-\sin 2\alpha}{-2\sin\alpha\cos\alpha} \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}$$

$=3$ ✓

$$\begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} \overbrace{\cos 2\alpha \cdot \cos\alpha - \sin 2\alpha \sin\alpha}^{\cos(2\alpha+\alpha)} & -\cos 2\alpha \sin\alpha - \sin 2\alpha \cos\alpha \\ \underbrace{\sin 2\alpha \cos\alpha + \cos 2\alpha \sin\alpha}_{\sin(2\alpha+\alpha)} & -\sin 2\alpha \sin\alpha + \cos 2\alpha \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos 3\alpha & -\sin 3\alpha \\ \sin 3\alpha & \cos 3\alpha \end{pmatrix}$$

Tfh. $n = k$-ra igaz!

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}^k = \begin{pmatrix} \cos k\alpha & -\sin k\alpha \\ \sin k\alpha & \cos k\alpha \end{pmatrix}$$

Biz be ↗, hogy $n = k+1$-re is igaz!

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}^{k+1} = \begin{pmatrix} \cos(k+1)\alpha & -\sin(k+1)\alpha \\ \sin(k+1)\alpha & \cos(k+1)\alpha \end{pmatrix}$$

$$\begin{pmatrix} \cos k\alpha & -\sin k\alpha \\ \sin k\alpha & \cos k\alpha \end{pmatrix} \begin{pmatrix} \cos k\alpha \cos\alpha - \sin k\alpha \sin\alpha & -\cos k\alpha \sin\alpha - \sin k\alpha \cos\alpha \\ \sin k\alpha \cos\alpha + \cos k\alpha \sin\alpha & \sin k\alpha \sin\alpha + \cos k\alpha \cos\alpha \end{pmatrix} \Rightarrow k+1.\ \text{hatvány} := A$$

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$A = \begin{pmatrix} \cos(k\alpha+\alpha) & -[\sin(k\alpha+\alpha)] \\ \sin(k\alpha+\alpha) & \cos(k\alpha+\alpha) \end{pmatrix} = \begin{pmatrix} \cos(k+1)\alpha & -\sin(k+1)\alpha \\ \sin(k+1)\alpha & \cos(k+1)\alpha \end{pmatrix}$$

Tehát igaz!

Determinánsok számítása

$$\begin{vmatrix} 3 & 5 \\ -1 & 4 \end{vmatrix} = 3\cdot 4 - (-1\cdot 5) = 12 + 5 = \underline{\underline{17}}$$

Alk. rá a Sarrus szabály

$$\begin{vmatrix} 2 & 5 \\ -3 & 6 \end{vmatrix} = 27$$

$$\begin{vmatrix} 2 & 9 & 4 \\ 3 & 5 & 1 \\ 1 & -3 & -6 \end{vmatrix} = 2\cdot 5(-6) + 3\cdot(-3)\cdot 4 + 9 - 5\cdot 4 - 2\cdot(-3) - 3\cdot 9\cdot(-6) = -60 - 36 + 9 - 20 + 6 + 162$$

$$\begin{matrix} 2 & 9 & 4 \\ 3 & 5 & 1 \end{matrix}$$

$$= \underline{\underline{61}}$$

$$\begin{vmatrix} 5 & 2 & 4 \\ 3 & -7 & 8 \\ -4 & 8 & 2 \end{vmatrix} = -70 + 96 - 64 - 112 - 320 - 12 = \underline{\underline{-482}}$$

$$\begin{matrix} 5 & 2 & 4 \\ 3 & -7 & 8 \end{matrix}$$

$$\begin{vmatrix} -2 & -3 & 4 \\ 5 & 2 & -3 \\ 4 & -1 & 5 \end{vmatrix} = \underbrace{-20 - 20 + 36}_{-4} \underbrace{-32 + 6 + 75}_{49} = \underline{\underline{45}}$$

$$\begin{matrix} -2 & -3 & 4 \\ 5 & 2 & -3 \end{matrix}$$

$$\begin{vmatrix} 1 & i & 1+i \\ -i & 1 & 0 \\ 1-i & 0 & 1 \end{vmatrix} = 1 - i + i^2 + i - i^2 - (1-i)(1+i) - 0 + i^2 =$$

$$\begin{matrix} 1 & i & 1+i \\ -i & 1 & 0 \end{matrix}$$

$$= \underline{\underline{-2}}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+b \end{vmatrix} = (1+a)(1+b) + 1 + 1 - 1 - a - 1 - 1 - b = 1 + b + a + ab - a - b - 1 = \underline{\underline{ab}}$$

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \end{matrix}$$

Hf.:

$$\begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & 3 \end{pmatrix} = A \qquad A^3 = ?$$

~o~

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 1 & 0 \end{pmatrix}_{2\times 3} \qquad \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 0 \end{pmatrix}_{3\times 2} = B \qquad AB = BA \; ?$$

~o~

$$A = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \qquad B = AA^T + 2A^T \cdot A$$

~o~

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{pmatrix} \qquad \begin{matrix} 2A + B \\ A^T + B^T \\ A^T B \end{matrix}$$

$$B = \begin{pmatrix} -1 & 5 & -2 \\ 1 & 1 & -1 \end{pmatrix}$$

~o~

$$\begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & 3 \end{pmatrix} = A$$

$$A = \begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & 3 \end{pmatrix} \qquad \begin{pmatrix} -5 & -6 & -42 \\ 9 & 10 & 63 \\ 8 & -4 & -3 \end{pmatrix} = A^2$$

$$\begin{pmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} -5 & -6 & -42 \\ 9 & 10 & 63 \\ 8 & -4 & -3 \end{pmatrix} \qquad \begin{pmatrix} -71 & -2 & -150 \\ 105 & 2 & 225 \\ 14 & -24 & -93 \end{pmatrix} = A^3$$

~o~

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 0 \end{pmatrix} = B$$

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 21 & 8 \\ 5 & 9 \end{pmatrix} = AB$$

$$\begin{pmatrix} 1 & 2 & 4 \\ 3 & 1 & 0 \end{pmatrix} = A$$

$$B = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 0 \end{pmatrix} \qquad \begin{pmatrix} 7 & 4 & 4 \\ 11 & 7 & 8 \\ 4 & 8 & 16 \end{pmatrix} = B \cdot A$$

~o~

$$\begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} = A^T$$

$$A = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} \qquad \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} = A \cdot A^T$$

$$\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix} = A$$

$$\begin{pmatrix} 5 & 10 \\ -4 & -2 \end{pmatrix} = 2A^T \qquad \begin{pmatrix} 25 & -20 \\ -8 & 10 \end{pmatrix} = 2A^T \cdot A$$

$$B = \begin{pmatrix} 30 & -16 \\ -4 & 15 \end{pmatrix}$$

~o~

$$2A + B = \begin{pmatrix} 1 & 9 & 4 \\ -1 & 1 & 3 \end{pmatrix} \qquad A^T + B^T = \begin{pmatrix} 0 & 0 \\ 7 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 5 & -2 \\ 1 & 1 & -1 \end{pmatrix} = B$$

$$A^T = \begin{pmatrix} 1 & -1 \\ 2 & 0 \\ 3 & 2 \end{pmatrix} \qquad \begin{pmatrix} -2 & 4 & -1 \\ -2 & 10 & -4 \\ -1 & 13 & -4 \end{pmatrix} A^T \cdot B$$

4. gyakorlat

11.24

$$\begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = \underline{\underline{1}}$$

$$\begin{matrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \end{matrix}$$

$$\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\begin{matrix} x^2 & 4 & 9 \\ x & 2 & 3 \end{matrix}$$

$$2x^2 + 9x + 12 - 18 - 3x^2 - 4x = 1$$

$$-x^2 + 5x - 6 = 0$$

$$0 = x^2 - 5x + 6$$

$$x_{12} = \frac{5 \pm \sqrt{25-24}}{2} = \dots$$

Hf.: $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0 \qquad x = ?$

Igazoljuk, h.

$$\begin{vmatrix} x & x+d & x+2d \\ y & y+d & y+2d \\ z & z+d & z+2d \end{vmatrix} = 0$$

$$\begin{matrix} x & x+d & x+2d \\ y & y+d & y+2d \end{matrix}$$

$$x(y+d)(z+2d) + y(z+d)(x+2d) + z(x+d)(y+2d) - z(y+d)(x+2d) - x(z+d)(y+2d) -$$

$$- y(x+d)(z+2d) = 0$$

Igazoljuk!

$$\begin{vmatrix} \sin 2x & -\cos 2x & 1 \\ \sin x & -\cos x & \cos x \\ \cos x & \sin x & \sin x \end{vmatrix} = 0$$

$$\sin 2x\cdot(-\cos x)\cdot\cos x + \sin x\cdot\sin x\cdot \quad + \cos x(-\cos 2x)\cos x - \cos x-(\cos x) - \sin 2x\cdot\sin x\cdot\cos x -$$

$$- \sin x\cdot\sin x\cdot(-\cos 2x) = 1 - 2(\sin 2x)(\cos x)(\sin x) - (\cos 2x)(\cos^2 x - \sin^2 x) =$$

$$= 1 - \underbrace{\sin^2 2x - \cos^2 2x}_{\text{Kiemeltünk mindkettőből } \ominus\text{-t.}} = 1 - 1 = 0$$

— o —

$$\begin{vmatrix} 1 & -3 & 2 \\ 4 & 3 & -5 \\ 2 & 1 & 0 \end{vmatrix} \overset{\text{1 sora szerint}}{=} 1\begin{vmatrix} 3 & -5 \\ 1 & 0 \end{vmatrix} - (-3)\begin{vmatrix} 4 & -5 \\ 2 & 0 \end{vmatrix} + 2\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$$

$+ \quad \cdot(-2)$

$$\begin{matrix} + & - & + & - & \dots \\ - & + & - & + & \dots \\ + & - & + & - & \dots \\ \vdots & \vdots & \vdots & \vdots & \end{matrix}$$

$$\begin{vmatrix} 1 & -3 & 2 \\ 4 & 3 & -5 \\ 2 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 7 & -3 & 2 \\ -2 & 3 & -5 \\ 0 & 1 & 0 \end{vmatrix} \overset{\text{3 sor szerint}}{=} 0\begin{vmatrix} -3 & 2 \\ 3 & -5 \end{vmatrix} - 1\begin{vmatrix} 7 & 2 \\ -2 & -5 \end{vmatrix} + 0\begin{vmatrix} 7 & -3 \\ -2 & 3 \end{vmatrix}$$

$\Downarrow$ 0 $\qquad \Downarrow$ $35 - 4 = 31$ $\qquad \Downarrow$

— o —

$$\begin{vmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{vmatrix} = \begin{array}{c} \begin{matrix} + & - & + & - \end{matrix} \\ \begin{vmatrix} 2 & 7 & -2 & 4 \\ 3 & 4 & 1 & 2 \\ 3 & 8 & 3 & 4 \\ -2 & 0 & 0 & 5 \end{vmatrix} \end{array} \overset{\text{4. sor sz.}}{=} 2\begin{vmatrix} 7 & -2 & 4 \\ 4 & 1 & 2 \\ 8 & 3 & 4 \end{vmatrix} + 5\begin{vmatrix} 2 & 7 & -2 \\ -3 & 4 & 1 \\ 3 & 8 & 3 \end{vmatrix}$$

1 oszlop ·2-vel
+ 2 oszlophoz

— o —

$$\begin{vmatrix} -1 & 2 & 5 & 4 \\ 1 & 3 & 4 & 5 \\ -1 & 4 & 3 & 2 \\ 0 & 5 & 2 & 3 \end{vmatrix} = \begin{vmatrix} -1 & 2 & 5 & 4 \\ 0 & 5 & 9 & 9 \\ -1 & 4 & 3 & 2 \\ 0 & 5 & 2 & 3 \end{vmatrix} \overset{\text{1. o. sz.}}{=} -1\begin{vmatrix} 2 & 5 & 4 \\ 5 & 9 & 9 \\ 4 & 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 5 & 4 \\ 5 & 9 & 9 \\ 5 & 2 & 3 \end{vmatrix} =$$

$$= 4 + 28 = 32$$

$$\begin{vmatrix} 5 & 1 & 2 & 7 \\ 3 & 0 & 0 & 2 \\ 1 & -3 & 4 & 5 \\ 2 & 0 & 0 & 3 \end{vmatrix} \overset{2.o.sz.}{=} -1\begin{vmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 0 & 3 \end{vmatrix} -3\begin{vmatrix} 5 & 2 & 7 \\ 3 & 0 & 2 \\ 2 & 0 & 3 \end{vmatrix} \overset{2.o.sz.}{=} -1(+4)\underbrace{\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}}_{5} -3(-2)\underbrace{\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}}_{5} =$$

$$= -20+30 = \underline{\underline{10}}$$

— o —

$$\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{vmatrix} = \begin{vmatrix} -2 & 1 & 0 & 1 \\ 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{vmatrix} \overset{3.o.sz.}{=} -1\begin{vmatrix} -2 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 2 & 0 \end{vmatrix} -1\begin{vmatrix} -2 & 1 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{vmatrix} = \underline{\underline{-12}}$$

2. sor $\cdot(-2)$-vel +1 sorhoz

— o —

$$\begin{vmatrix} 1 & 0 & 0 & -1 \\ 2 & 3 & 4 & 7 \\ -3 & 4 & 5 & 9 \\ -4 & -5 & 6 & 1 \end{vmatrix} \overset{1. sor sz.}{=} \begin{vmatrix} 3 & 4 & 7 \\ 4 & 5 & 9 \\ -5 & 6 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 3 & 4 \\ -3 & 4 & 5 \\ -4 & -5 & 6 \end{vmatrix} = \underline{\underline{216}}$$

— o —

Bedüt tart. sor neriul fejtsük ki!

$$\begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ a & b & c & d \\ -1 & -1 & 1 & 0 \end{vmatrix} = a\begin{vmatrix} 0 & -1 & -1 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \end{vmatrix} - b\begin{vmatrix} 1 & -1 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{vmatrix} + c\begin{vmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix} - d\begin{vmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{vmatrix} =$$

$$= 3a - b + 2c + d$$

— o —

$$\begin{vmatrix} 2 & 1 & 1 & x \\ 1 & 2 & 1 & y \\ 1 & 1 & 2 & z \\ 1 & 1 & 1 & t \end{vmatrix} = -x\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} + y\begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} - z\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + t\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} =$$

$$= -x - y - z + 4t$$

— o —

$$\begin{vmatrix} a & 1 & 1 & 1 \\ b & 0 & 1 & 1 \\ c & 1 & 0 & 1 \\ d & 1 & 1 & 0 \end{vmatrix} = a\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} - b\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} + c\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} - d\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} =$$

$$= 2a - b - c - d$$

Hf.: $\begin{vmatrix} -4 & -1 & 2 & 3 & 1 \\ 2 & 3 & 0 & 4 & -1 \\ 0 & -3 & 0 & 5 & 3 \\ -1 & -1 & 0 & 1 & -1 \\ 0 & -4 & -2 & -1 & -2 \end{vmatrix}$ -48

$$\begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 & 0 \\ 3 & 6 & 1 & 0 & 0 & 0 \\ 4 & 9 & 14 & 1 & 1 & 1 \\ 5 & 12 & 24 & 1 & 5 & 9 \\ 9 & 24 & 38 & 1 & 25 & 81 \end{vmatrix}$$

$$\begin{vmatrix} -4 & -1 & 2 & 3 & 1 \\ 2 & 3 & 0 & 4 & -1 \\ 0 & -3 & 0 & 5 & 3 \\ -1 & -1 & 0 & 1 & -1 \\ 0 & -4 & -2 & -1 & -2 \end{vmatrix} \overset{3.o.}{=} 2\begin{vmatrix} 2 & 3 & 4 & -1 \\ 0 & -3 & 5 & 3 \\ -1 & -1 & 1 & -1 \\ 0 & -4 & -1 & -2 \end{vmatrix} - 2\begin{vmatrix} -4 & -1 & 3 & 1 \\ 2 & 3 & 4 & -1 \\ 0 & 3 & 5 & 3 \\ -1 & -1 & 1 & -1 \end{vmatrix} \overset{1o.}{=} 2\left(2\begin{vmatrix} -3 & 5 & 3 \\ -1 & 1 & -1 \\ -4 & -1 & -2 \\ -3 & 5 & 3 \\ -1 & 1 & -1 \end{vmatrix} - \begin{vmatrix} 3 & 4 & -1 \\ -3 & 5 & 3 \\ -4 & -1 & -2 \\ 3 & 4 & -1 \\ -3 & 5 & 3 \end{vmatrix} \right) -$$

$$-2\left(-4\begin{vmatrix} 3 & 4 & -1 \\ 3 & 5 & 3 \\ -1 & 1 & -1 \\ 3 & 4 & -1 \\ 3 & 5 & 3 \end{vmatrix} - 2\begin{vmatrix} -1 & 3 & 1 \\ 3 & 5 & 3 \\ -1 & 1 & -1 \\ -1 & 3 & 1 \\ 3 & 5 & 3 \end{vmatrix} + \begin{vmatrix} -1 & 3 & 1 \\ 3 & 4 & -1 \\ 3 & 5 & 3 \\ -1 & 3 & 1 \\ 3 & 4 & -1 \end{vmatrix} \right) = 2(84+116) - 2(-8-32-50) =$$

$$= 580$$

$$\underset{5.o.-4o.}{\begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 & 0 \\ 3 & 6 & 1 & 0 & 0 & 0 \\ 4 & 9 & 14 & 1 & 1 & 1 \\ 5 & 12 & 24 & 1 & 5 & 9 \\ 9 & 24 & 38 & 1 & 25 & 81 \end{vmatrix}} = \underset{5.\text{sor}\cdot 6 \text{ és } \ominus 6.\text{sor}}{\begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 & 0 \\ 3 & 6 & 1 & 0 & 0 & 0 \\ 4 & 9 & 14 & 1 & 0 & 1 \\ 5 & 12 & 24 & 1 & 4 & 9 \\ 9 & 24 & 38 & 1 & 24 & 81 \end{vmatrix}} = \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 & 0 \\ 3 & 6 & 1 & 0 & 0 & 0 \\ 4 & 9 & 14 & 1 & 0 & 1 \\ 5 & 12 & 24 & 1 & 4 & 9 \\ -27 & -48 & -106 & -5 & 0 & -27 \end{vmatrix} \overset{5.o.}{=} 4\begin{vmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 \\ 3 & 6 & 1 & 0 & 0 \\ 4 & 9 & 14 & 1 & 1 \\ -27 & -48 & -106 & -5 & -27 \end{vmatrix} =$$