analízis 1.

7. gyakorlat

XI. 11.

1, $\int \frac{\cos 2x}{\sin^2 2x}\,dx = \int \cos 2x \cdot (\sin 2x)^{-2}\,dx = \frac{1}{2}\int 2\cos 2x\,(\sin 2x)^{-2}\,dx = *$

$(\sin 2x)' = \cos 2x \cdot 2$

$* = \frac{1}{2}\,\frac{(\sin 2x)^{-2+1}}{-2+1} + c = \frac{-1}{2\sin 2x} + c$

2, $\int \frac{1+\cos x - \cos^3 x}{\sin^2 x}\,dx = \int \frac{1+\cos x\,(1-\cos^2 x)}{\sin^2 x}\,dx =$

$= \int \frac{1+\cos x \sin^2 x}{\sin^2 x}\,dx = \int \left(\frac{1}{\sin^2 x} + \cos x\right)dx = -\operatorname{ctg} x + \sin x + c$

3, $\int \frac{\sin 2x}{1+\sin^2 x}\,dx = \int \frac{(1+\sin^2 x)'}{1+\sin^2 x}\,dx = \ln|1+\sin^2 x| + c$

4, $\int \frac{1}{\sin 2x}\,dx = \int \frac{\sin^2 x + \cos^2 x}{2\sin x\cos x}\,dx = \frac{1}{2}\int\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)dx = \frac{1}{2}\int\Big(-\frac{-\sin x}{\cos x} +$

$+ \frac{\cos x}{\sin x}\Big)dx = \frac{1}{2}\int\left(-\frac{(\cos x)'}{\cos x} + \frac{(\sin x)'}{\sin x}\right)dx = \frac{1}{2}\big(-\ln|\cos x| +$

$+ \ln|\sin x|\big) + c = \frac{1}{2}\cdot\ln|\operatorname{tg} x| + c = \ln\sqrt{|\operatorname{tg} x|} + c$

5, $\int \frac{1}{x+\sqrt{x}}\,dx = \int \left(\frac{1}{\sqrt{x}(\sqrt{x}^2+1)}\right)dx = 2\int \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}+1} = 2\int\frac{(\sqrt{x}+1)'}{(\sqrt{x}+1)}\,dx =$

$= 2\ln|\sqrt{x}+1| + c = \ln(\sqrt{x}+1)^2 + c = \ln(x+2\sqrt{x}+1) + c$

$(\sqrt{x}+1)' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

6, $\int \frac{1}{\sin 2x}\,dx = \int \frac{1}{2\sin x\cos x}\,dx = \frac{1}{2}\int\frac{1}{\sin x\cos x}\,dx =$

$= \frac{1}{2}\int \frac{\frac{1}{\cos x}}{\sin x}\,dx = \frac{1}{2}\int\frac{\frac{1}{\cos^2 x}}{\operatorname{tg} x}\,dx = \frac{1}{2}\int\frac{(\operatorname{tg} x)'}{\operatorname{tg} x}\,dx = \frac{1}{2}\ln|\operatorname{tg} x| + c$

7,

$$\int \frac{1}{x+2\sqrt[3]{x}}\,dx = \int\left(\frac{1}{x}+\frac{1}{2\sqrt[3]{x}}\right)dx = \int \frac{1}{x}\,dx + \int\left(\frac{1}{2}\cdot\frac{1}{\sqrt[3]{x}}\right)dx =$$

$x^{-\frac{1}{3}}$

$2\sqrt[3]{x} = 2\cdot x^{\frac{1}{3}}$

$x+2\sqrt[3]{x}$

$\sqrt[3]{x^2}$ $\quad \sqrt[3]{x}\left(\sqrt[3]{x^2}+2\right)$

$$= \ln|x| + \frac{1}{2}\;\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C = \ln|x|$$

$$\int \frac{1}{x+2\sqrt[3]{x}}\,dx = \int \frac{1}{\sqrt[3]{x}\left(\sqrt[3]{x^2}+2\right)}\,dx = \int \frac{\frac{2}{3\cdot\sqrt[3]{x}}}{x^{\frac{2}{3}}+2}\,dx = \frac{3}{2}\int \frac{\left(x^{\frac{2}{3}}+2\right)'}{x^{\frac{2}{3}}+2} = *$$

$$\left(x^{\frac{2}{3}}+2\right)' = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\cdot\sqrt[3]{x}}$$

$$* = \underline{\underline{\frac{3}{2}\ln\left|x^{\frac{2}{3}}+2\right| + C}}$$

8,

$$\int \frac{e^x+1}{e^x+e^{-x}+2}\,dx = \int \frac{e^x+1}{\frac{(e^x+1)^2}{e^x}}\,dx = \int\left(\cancel{(e^x+1)}\cdot\frac{e^x}{(e^x+1)^{\cancel{2}}}\right)dx = \int\frac{e^x}{e^x+1}\,dx = \int\frac{(e^x+1)'}{(e^x+1)}\,dx = *$$

$$e^x + \frac{1}{e^x} + 2 = \frac{e^{2x}+2e^x+1}{e^x}$$

$$(e^x+1)' = e^x$$

$$* = \underline{\underline{\ln|e^x+1| + C}}$$

9,

$$\int \frac{1}{x\ln^2 x\cdot\log_x 2}\,dx = \int \frac{1}{x\ln x\ln 2}\,dx = \frac{1}{\ln 2}\int \frac{1}{x\ln x}\,dx = \frac{1}{\ln 2}\int \frac{\frac{1}{x}}{\ln x}\,dx = *$$

$$\frac{1}{x\ln^2 x\cdot\frac{\ln 2}{\ln x}}$$

$x\cdot\ln x\cdot\ln 2$

$x \cdot \ln(x+2)$

$$* = \frac{1}{\ln 2}\int\frac{(\ln x)'}{\ln x}\,dx = \underline{\underline{\frac{1}{\ln 2}\ln|\ln x| + C}}

$$= \int \frac{e^x(e^x+1)}{(e^x+1)^2}dx = \int \frac{e^x}{e^x+1}dx = \int \frac{(e^x+1)'}{e^x+1}dx = \ln|e^x+1| + c$$

9.) $$\int \frac{1}{x\cdot \ln^2 x \cdot \log_x 2}dx = \int \frac{1}{x\cdot \ln^2 x\cdot \frac{\ln 2}{\ln x}} = \int \frac{1}{x\cdot \ln x\cdot \ln 2}dx =$$

$$= \frac{1}{\ln 2}\int \frac{1}{x\cdot \ln x}dx = \frac{1}{\ln 2}\int \frac{\frac{1}{x}}{\ln x}dx = \frac{1}{\ln 2}\int \frac{(\ln x)'}{\ln x}dx =$$

$$= \frac{1}{\ln 2}\cdot \ln|\ln x| + c$$

Parciális integrálás: XI. 18.

$$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + g'(x)\cdot f(x)$$

$$\int (f'(x)g(x) + g'(x)f(x))dx = f(x)\cdot g(x) + c$$

$$\boxed{\int (f'(x)g(x))dx = f(x)g(x) - \int f(x)\cdot g'(x)dx}$$

Pl: 1.) $\int x\cdot e^x dx =$

$f'(x) = e^x$ $\quad f(x) = e^x$

$g(x) = x$ $\quad g'(x) = 1$

$$\int x\cdot e^x dx = x\cdot e^x - \int e^x\cdot 1\, dx = xe^x - e^x + c$$

2.) $$\int \ln x\, dx = \int 1\cdot \ln x\, dx = x\cdot \ln x - \int \underbrace{x\cdot \frac{1}{x}}_{1}dx = x\cdot \ln x - x + c$$

$f'(x) = 1$ $\quad f(x) = x$

$g(x) = \ln x$ $\quad g'(x) = \frac{1}{x}$

Hf: $\int \log_2 x\, dx$; $\int \log_a x\, dx$ $(a>0,\ a\neq 1)$

3.) $$\int \arcsin x\, dx = \int 1\cdot \arcsin x\, dx = x\cdot \arcsin x - \int \frac{x}{\sqrt{1-x^2}}dx =$$

$f'(x) = 1$ $\quad f(x) = x$

$g(x) = \arcsin x$ $\quad g'(x) = \frac{1}{\sqrt{1-x^2}}$

$$\left(\frac{x}{\sqrt{1-x^2}} = x(1-x^2)^{-\frac{1}{2}} = -\frac{1}{2}(-2x)(1-x^2)^{-\frac{1}{2}} = -\frac{1}{2}(1-x^2)'(1-x^2)^{-\frac{1}{2}}\right)$$

$$= x\arcsin x + \frac{1}{2}\int (1-x^2)'(1-x^2)^{-\frac{1}{2}}dx = x\cdot \arcsin x + \frac{1}{2}\cdot \frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

Hf: $\int \arccos x\, dx$

4.) $$\int \operatorname{arctg} x\, dx = \int 1\cdot \operatorname{arctg} x\, dx = x\cdot \operatorname{arctg} x - \int \frac{x}{1+x^2} =$$

$f'(x) = 1$ $\quad f(x) = x$

$g(x) = \operatorname{arctg} x$ $\quad g'(x) = \frac{1}{1+x^2}$

$$\left(\frac{x}{1+x^2} = x\cdot \frac{1}{1+x^2} = \frac{1}{2}\cdot \frac{2x}{1+x^2} = \frac{1}{2}\cdot \frac{(1+x^2)'}{1+x^2}\right)$$

$$= x\cdot \operatorname{arctg} x - \frac{1}{2}\int \frac{(1+x^2)'}{1+x^2}dx = x\cdot \operatorname{arctg} x - \frac{1}{2}\ln|1+x^2| + c$$

Hf: $\int \operatorname{arcctg} x\, dx$

5.) $$\int e^x\cdot \sin x\, dx = e^x\cdot \sin x - \int e^x \cos x\, dx =$$

$f'(x) = e^x$ $\quad f(x) = e^x$ $\quad | \quad f'(x) = e^x$ $\quad f(x) = e^x$

$g(x) = \sin x$ $\quad g'(x) = \cos x$ $\quad | \quad g(x) = \cos x$ $\quad g'(x) = -\sin x$

$$= e^x\sin x - \left(e^x\cos x - \int e^x(-\sin x)dx\right) = e^x \sin x - e^x\cos x -$$

$- \int e^x\sin x\, dx$ Ez egy egyenlet

$$2\cdot \int e^x \sin x = e^x\sin x - e^x\cos x + \underbrace{2c}$$

→ hogy szépen nézzen ki osztás után

$$\int e^x\sin x = \frac{1}{2}\cdot e^x(\sin x - \cos x) + c$$

6.) $$\int \frac{x^3}{\sqrt{1+x^2}}dx = \int x^2\cdot \frac{x}{\sqrt{1+x^2}}dx$$

$f'(x) = \frac{x}{\sqrt{1+x^2}}$ $\quad f(x) = \int \frac{x}{\sqrt{1+x^2}}dx = \frac{1}{2}\int 2x(1+x^2)^{-\frac{1}{2}}dx =$

$g(x) = x^2$ $\quad g'(x) = 2x$

$f(x) = \frac{1}{2}\int (1+x^2)'(1+x^2)^{-\frac{1}{2}}dx = \frac{1}{2}\cdot\frac{\sqrt{1+x^2}}{\frac{1}{2}} + C = \sqrt{1+x^2} + C$

$= x^2\cdot\sqrt{1+x^2} - \int 2x\sqrt{1+x^2}\,dx = x^2\sqrt{1+x^2} - \int(1+x^2)'(1+x^2)^{\frac{1}{2}}dx =$

$= x^2\cdot\sqrt{1+x^2} - \frac{(1+x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C$

7.) $\int \cos \ln x\,dx = x\cdot\cos\ln x - \int x(-\sin\ln x)\frac{1}{x}dx =$

$f'(x) = 1 \qquad f(x) = x$

$g(x) = \cos\ln x \qquad g'(x) = -\sin\ln x\cdot\frac{1}{x}$

$= x\cdot\cos\ln x + \int\sin\ln x\,dx \quad = x\cdot\cos\ln x + x\cdot\sin\ln x -$

$f'(x) = 1 \qquad f(x) = x$

$g(x) = \sin\ln x \quad g'(x) = \cos\ln x\cdot\frac{1}{x}$

$- \int\cos\ln x\,dx$

$2\int\cos\ln x\,dx = x\cdot\cos\ln x + x\sin\ln x + 2c$

$\int\cos\ln x\,dx = \frac{1}{2}x\,(\cos\ln x + \sin\ln x) + c$

8.) $\int\frac{\arcsin x}{\sqrt{x+1}}dx = \int\arcsin x\cdot\frac{1}{\sqrt{x+1}}dx =$

$f'(x) = \frac{1}{\sqrt{x+1}} \qquad f(x) = \int\frac{1}{\sqrt{x+1}}dx = \int 1\cdot(x+1)^{-\frac{1}{2}}dx =$

$= \int(x+1)'\cdot(x+1)^{-\frac{1}{2}}dx = 2\sqrt{x+1} + c$

$g(x) = \arcsin x$

$g'(x) = \frac{1}{\sqrt{1-x^2}}$

$= 2\sqrt{x+1}\cdot\arcsin x - \int 2\sqrt{x+1}\cdot\frac{1}{\sqrt{1-x^2}}dx =$

$2\sqrt{x+1}\cdot\frac{1}{\sqrt{1-x^2}} = 2\sqrt{\frac{x+1}{1-x^2}} = 2\sqrt{\frac{1}{1-x}} = \frac{2}{\sqrt{1-x}}$

$= 2\sqrt{x+1}\arcsin x + 2\int(-1)(1-x)^{-\frac{1}{2}}dx = 2\sqrt{x+1}\arcsin x + 2\frac{(1-x)^{\frac{1}{2}}}{\frac{1}{2}} + C$

9.) $\int e^x dx = e^x + c$

$(e^{f(x)} + c)' = e^{f(x)}\cdot f'(x) \Rightarrow \boxed{\int f'(x)e^{f(x)}dx = e^{f(x)} + c}$

10.) $\int\frac{1}{e^x}dx = \int e^{-x}dx = -\int -e^{-x}dx = -\int(-x)'\cdot e^{-x}dx = -e^{-x} + c$

11.) $\int\sin x\,dx = -\cos x + c$

$\boxed{\int f'(x)\cdot\sin f(x)\,dx = -\cos f(x) + c}$

12.) $\int\sin(2x)dx = \int 2\cdot\sin x\cos x\,dx = 2\int(\sin x)'\cdot(\sin x)^1 dx = 2\cdot\frac{\sin^2 x}{2} + c$

vagy

ez alapján:

$\int\sin(2x)dx = \frac{1}{2}\int\underbrace{(2x)'}_{2}\sin(2x)dx = -\frac{1}{2}\cos(2x) + c$

13.) $\int\sin(\pi x)dx = \frac{1}{\pi}\int\underbrace{(\pi x)'}_{\pi}\cdot\sin(\pi x)\,dx = -\frac{1}{\pi}\cdot\cos(\pi x) + c$

14.) $\int\frac{1}{\sqrt{1-x^2}}dx = \arcsin x + c$

$\boxed{\int\frac{f'(x)}{\sqrt{1-f^2(x)}}dx = \arcsin f(x) + c}$

Pl: ~~$\int \operatorname{ctg} x\,dx = \int\frac{\cos x}{\sin x}dx = \int(\sin x)'$~~

$\int\frac{1}{\sqrt{1-4x^2}}dx = \int\frac{1}{\sqrt{1-(2x)^2}}dx = \frac{1}{2}\int\frac{2}{\sqrt{1-(2x)^2}}dx = \frac{1}{2}\arcsin(2x) + c$

15.) $\int\frac{e^x}{\sqrt{1-e^{2x}}}dx = \int\frac{e^x}{\sqrt{1-(e^x)^2}}dx = \int\frac{(e^x)'}{\sqrt{1-(e^x)^2}}dx = \arcsin e^x + c$

16.) $\int\frac{1}{\sqrt{9-x^2}}dx = \int\frac{1}{\sqrt{9(1-\frac{x^2}{9})}}dx = \frac{1}{3}\int\frac{1}{\sqrt{1-\frac{x^2}{9}}}dx = \frac{1}{3}\int\frac{1}{\sqrt{1-(\frac{x}{3})^2}}dx =$

$= \frac{1}{3}\arcsin\left(\frac{x}{3}\right) + c$

HC: 1.) $\int \frac{1}{\sqrt{2x-x^2}}\,dx$ 3.) $\int \frac{\cos \ln x}{x}\,dx$

2.) $\int \frac{x^2}{\sqrt{1-x^6}}\,dx$ 4.) $\int \operatorname{ctg} x \cdot \sqrt{\sin x}\,dx$

HC::

1) $\int \log_3 x\,dx = \int 1\cdot \log_3 x\,dx = x\cdot\log_3 x - \int \cancel{x}\cdot\frac{1}{\cancel{x}\cdot \ln 3}\,dx =$

$f'(x)=1 \quad f(x)=x$

$g(x)=\log_3 x \quad g'(x) = \frac{1}{x\cdot \ln 3}$

$= x\cdot \log_3 x - \frac{x}{\ln 3} + C$

2.) $\int \log_a x\,dx = \int 1\cdot \log_a x\,dx = x\cdot\log_a x - \int \cancel{x}\cdot\frac{1}{\cancel{x}\ln a}\,dx = x\cdot \log_a x - \frac{x}{\ln a} + C$

$f'(x)=1 \quad f(x)=x$

$g(x)=\log_a x \quad g'(x)=\frac{1}{x\cdot \ln a}$

3.) $\int \arccos x\,dx = \int 1\cdot \arccos x\,dx = x\cdot\arccos x + \int \frac{x}{(1-x^2)^{\frac{1}{2}}}\,dx =$

$f'(x)=1 \quad f(x)=x$

$g(x)=\arccos x \quad g'(x)=\frac{-1}{\sqrt{1-x^2}}$

~~$= x\cdot\arccos x - \frac{1}{2}\int\frac{-2x}{(1-x^2)^{\frac{1}{2}}}\,dx = x\cdot\arccos x - \frac{1}{2}\ln|1-x^2| + C$~~

$= x\cdot\arccos x + \int x\cdot(1-x^2)^{-\frac{1}{2}}\,dx = x\cdot\arccos x - \frac{1}{2}\int(-2x)(1-x^2)^{-\frac{1}{2}}\,dx =$

$= x\cdot\arccos x - \frac{1}{2}\cdot\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + C$

4.) $\int \operatorname{arcctg} x\,dx = \int 1\cdot\operatorname{arcctg} x\,dx = x\cdot\operatorname{arcctg} x + \int\frac{x}{1+x^2}\,dx =$

$f'(x)=1 \quad f(x)=x$

$g(x)=\operatorname{arcctg} x \quad g'(x)=\frac{-1}{1+x^2}$

$= x\cdot\operatorname{arcctg} x + \frac{1}{2}\int\frac{2x}{1+x^2}\,dx = x\cdot\operatorname{arcctg} x + \frac{1}{2}\ln|1+x^2| + C$

5.) $\int\frac{1}{(2x-x^2)^{\frac{1}{2}}}\,dx = \int 1\cdot\frac{1}{(2x-x^2)^{\frac{1}{2}}}\,dx = \frac{x}{(2x-x^2)^{\frac{1}{2}}} + \frac{1}{2}\int x\cdot\frac{(2-2x)}{(2x-x^2)^{\frac{3}{2}}}\,dx =$

$f'(x)=1 \quad f(x)=x$

$g(x)=(2x-x^2)^{-\frac{1}{2}} \quad g'(x)=-\frac{1}{2}(2x-x^2)^{-\frac{3}{2}}\cdot(2-2x)$

$\frac{1}{2}\int x\cdot\frac{(2-2x)}{((2x-x^2)^2)^{\frac{3}{4}}}$

~~$\frac{1}{\sqrt{2x(1-\frac{x}{2})}} = \frac{1}{\sqrt{2}\cdot\sqrt{x}\cdot\sqrt{1-\frac{x}{2}}} = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{x}}\cdot\frac{1}{\sqrt{1-\frac{x}{2}}}$~~

~~$\frac{1}{\sqrt{2}}\int\frac{1}{\sqrt{x}}\cdot\frac{1}{\sqrt{1-(\frac{x}{\sqrt{2}})^2}}\,dx$~~

~~$f'(x)=\frac{1}{\sqrt{1-(\frac{x}{\sqrt{2}})^2}} \quad f(x)=\arcsin\left(\frac{x}{\sqrt{2}}\right)$~~ $f'(x)=x^{-\frac{1}{2}} \quad f(x)=2\cdot x^{\frac{1}{2}}$

~~$g(x)=\frac{1}{\sqrt{x}}=x^{-\frac{1}{2}} \quad g'(x)=-\frac{1}{2}\cdot x^{-\frac{3}{2}}$~~ $g(x)=\left(1-\frac{x}{2}\right)^{\frac{1}{2}} \quad g'(x)=\frac{1}{2}\left(1-\frac{x}{2}\right)^{-\frac{1}{2}}\cdot\left(-\frac{1}{2}x\right)$

$\int\frac{1}{\sqrt{2x(1-\frac{x}{2})}}\,dx = \frac{1}{\sqrt{2}}\int\frac{1}{\sqrt{x}}\cdot\frac{1}{\sqrt{1-\frac{x}{2}}}\,dx = \frac{1}{\sqrt{2}}\int\left(x^{-\frac{1}{2}}\right)\cdot\left(1-\frac{x}{2}\right)^{-\frac{1}{2}}\,dx$

6.) $\int\cos\ln x\cdot\frac{1}{x}\,dx = \ln x\cdot\cos\ln x + \int\ln x\cdot\sin\ln x\cdot\frac{1}{x}$

$f'(x)=\frac{1}{x} \quad f(x)=\ln x$ ~~$g(x)=\cos\ln x$~~

$g(x)=\cos\ln x \quad g'(x)=-\sin\ln x\cdot\frac{1}{x}$

5.) $\int 1\cdot(2x-x^2)^{-\frac{1}{2}}\,dx =$

~~$f'(x)=(2x-x^2)^{-\frac{1}{2}}$~~ ~~$f(x)=2\cdot(2x-x^2)^{\frac{1}{2}}$~~

$= \frac{x}{(2x-x^2)^{\frac{1}{2}}} + \frac{1}{2}\int x\cdot\frac{(2x-x^2)'}{(2x-x^2)^{\frac{3}{2}}}\,dx$

6.) $\int\frac{\ln x}{x}\cdot\sin\ln x\,dx$

5.) ~~$\int\frac{1}{\sqrt{x}\sqrt{2-x}}\,dx$~~ $(x-1)^2 = x^2-2x+1 \Rightarrow 2x-x^2 = 1-(x-1)^2$

$\int\frac{1}{\sqrt{1-(x-1)^2}}\,dx = \int\frac{(x-1)'}{\sqrt{1-(x-1)^2}}\,dx = \arcsin(x-1)+C$

android . uw. hu

9 . gyakorlat

XI. 25.

1, $\int \frac{1}{\sqrt{2x-x^2}}\,dx = \cancel{\int \frac{1}{\sqrt{x(2-x)}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{2-x}}\,dx =}$

$= \int \frac{1}{\sqrt{1-(x-1)^2}}\,dx = \int \frac{(x-1)'}{\sqrt{1-(x-1)^2}} = \underline{\underline{\arcsin(x-1)+C}}$

$f(x) = x-1$

$f'(x) = 1$

$\boxed{\int \frac{f'(x)}{\sqrt{1-f^2(x)}}\,dx = \arcsin f(x) + C}$

2, $\int \frac{\cos \ln x}{x}\,dx = \int \ln x' \cdot \cos(\ln x)\,dx = \underline{\underline{\sin(\ln x)+C}}$

$(\ln x)' = \frac{1}{x}$

$\boxed{\int f'(x) \cos f(x)\,dx = \sin f(x) + C}$

3, $\int \frac{x^2}{\sqrt{1-x^6}}\,dx = \int \frac{x^2}{\sqrt{1-(x^3)^2}}\,dx = \frac{1}{3}\int \frac{3x^2}{\sqrt{1-(x^3)^2}}\,dx = \underline{\underline{\frac{1}{3} \cdot \arcsin x^3 + C}}$

$(x^3)' = (3x^2)$

4, $\int \operatorname{ctg} x \cdot \sqrt{\ln \sin x}\,dx = \int (\ln \sin x)' \cdot (\ln \sin x)^{\frac{1}{2}}\,dx = \underline{\underline{\frac{(\ln \sin x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C}}$

$(\ln \sin x)' = \frac{1}{\sin x} \cdot \cos x = \operatorname{ctg} x$

5,

$$\int \sqrt{x} \cdot \operatorname{arctg}\sqrt{x}\, dx =$$

$$f'(x) = \sqrt{x} \qquad f(x) = \int \sqrt{x}\, dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \overset{0}{\tilde{C}} = \frac{2}{3} \cdot x^{\frac{3}{2}}$$

$$g(x) = \operatorname{arctg}\sqrt{x} \qquad g'(x) = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{2}{3} x^{\frac{3}{2}} \operatorname{arctg}\sqrt{x} - \int \frac{2}{3} x^{\frac{3}{2}} \cdot \frac{1}{1+x} \cdot \frac{1}{2x^{\frac{1}{2}}}\, dx = \frac{2}{3} x^{\frac{3}{2}} \operatorname{arctg}\sqrt{x} - \frac{1}{3} \int \frac{x}{1+x}\, dx$$

$$= \frac{2}{3} x^{\frac{3}{2}} \cdot \operatorname{arctg}\sqrt{x} - \frac{1}{3} \int \frac{(x+1)-1}{1+x}\, dx = \frac{2}{3} x^{\frac{3}{2}} \cdot \operatorname{arctg}\sqrt{x} - \frac{1}{3} \int 1 - \frac{\overset{1}{\overset{\shortparallel}{(1+x)}}}{1+x}\, dx =$$

$$= \frac{2}{3} x^{\frac{3}{2}} \cdot \operatorname{arctg}\sqrt{x} - \frac{1}{3} \left(x - \ln|1+x| \right) + C$$

$$f(x) \cdot g(x) - \int f(x) \cdot g'(x)$$

6,

$$\int \frac{\arcsin x}{\sqrt{x+1}}\, dx = \int \arcsin x \cdot (x+1)^{-\frac{1}{2}} =$$

$$f'(x) = (x+1)^{-\frac{1}{2}} \qquad f(x) = \int (x+1)^{-\frac{1}{2}} = \frac{(x+1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{(x+1)^{\frac{1}{2}}}{0{,}5} = 2 \cdot (x+1)^{\frac{1}{2}}$$

$$g(x) = \arcsin x \qquad g'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$= 2(x+1)^{+\frac{1}{2}} \cdot \arcsin x - \int 2(x+1)^{+\frac{1}{2}} \cdot \frac{1}{\sqrt{1-x^2}}\, dx = 2(x+1)^{+\frac{1}{2}} \arcsin x - \int 2\sqrt{x+1} \cdot \frac{1}{\sqrt{1-x^2}}\, dx =$$

$$= 2\sqrt{x+1} \arcsin x - \int \frac{2\sqrt{x+1}}{(1-x)(1+x)}\, dx = 2\sqrt{x+1} \arcsin x - 2 \int \frac{1}{\sqrt{1-x}}\, dx =$$

$$= 2\sqrt{x+1} \arcsin x + 2 \int (1-x)' (1-x)^{-\frac{1}{2}}\, dx = 2\sqrt{x+1} \arcsin x + 2 \frac{(1-x)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

I)

$$\int \frac{1}{x^2+8x+20}\,dx = \int \frac{1}{(x+4)^2-16+20}\,dx = \int \frac{1}{(x+4)^2+4} =$$

$$\int \frac{f'(x)}{1+f^2(x)}\,dx = \operatorname{arctg} x + c \qquad f'(x) = \left(\frac{x+4}{2}\right)' = \frac{1}{2}$$

$$= \frac{1}{4}\int \frac{1}{1+\frac{(x+4)^2}{4}}\,dx = \frac{1}{4}\int \frac{1}{1+\left(\frac{x+4}{2}\right)^2}\,dx = \frac{1}{4}\cdot 2\int \frac{\frac{1}{2}}{1+\left(\frac{x+4}{2}\right)^2}\,dx =$$

$$= \frac{1}{2}\int \frac{\left(\frac{x+4}{2}\right)'}{1+\left(\frac{x+4}{2}\right)^2}\,dx = \underline{\underline{\frac{1}{2}\operatorname{arctg}\left(\frac{x+4}{2}\right)+c}}$$

Ez a módszer ilyen alakban mindig alkalmazható!

γ;

$$\int \frac{1}{x^2+3x+4}\,dx = \int \frac{1}{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+4} = \int \frac{1}{\frac{7}{4}+\left(x+\frac{3}{2}\right)^2}\,dx =$$

$$= \frac{4}{7}\int \frac{1}{1+\frac{\left(x+\frac{3}{2}\right)^2}{\frac{7}{4}}}\,dx = \frac{4}{7}\int \frac{1}{1+\left(\frac{x+\frac{3}{2}}{\sqrt{\frac{7}{4}}}\right)^2} = \frac{4}{7}\cdot\frac{\sqrt{7}}{2}\int \frac{\frac{2}{\sqrt{7}}}{1+\left(\frac{2x+3}{\sqrt{7}}\right)^2}\,dx =$$

$$\left(\frac{x+\frac{3}{2}}{\sqrt{\frac{7}{4}}}\right)' = \left(\frac{2x+3}{\sqrt{7}}\right)' = \frac{2}{\sqrt{7}}$$

$$= \frac{2}{\sqrt{7}}\int \frac{\left(\frac{2x+3}{\sqrt{7}}\right)'}{1+\left(\frac{2x+3}{\sqrt{7}}\right)^2}\,dx = \underline{\underline{\frac{2}{\sqrt{7}}\operatorname{arctg}\left(\frac{2x+3}{\sqrt{7}}\right)+c}}$$

9, $\int \frac{1}{x^2-9}\,dx = \int \frac{1}{(x-3)(x+3)}\,dx = \int \left(\frac{\frac{1}{6}}{x-3} - \frac{\frac{1}{6}}{x+3}\right) dx =$

Parciális (elemi) törtekre való bontás.

MINDIG LEHET $\frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3} = \frac{A(x+3) + B(x-3)}{(x-3)(x+3)} =$

$= \frac{\overbrace{(A+B)}^{0}x + \overbrace{(3A-3B)}^{1}}{(x-3)(x+3)}$ $\forall x - re$

$A+B=0 \quad B=-A$

$3A-3B=1$

$3A+3A=1$

$A=\frac{1}{6} \Rightarrow B=-\frac{1}{6}$

$D>0$

$= \frac{1}{6}\int\left(\frac{1}{x-3} - \frac{1}{x+3}\right)dx = \frac{1}{6}\int\left(\frac{(x-3)'}{x-3} - \frac{(x+3)'}{x+3}\right)dx = \frac{1}{6}\left(\ln|x-3| - \ln|x+3|\right) + C =$

$= \ln\sqrt[6]{\left|\frac{x-3}{x+3}\right|} + C$

10, $\int \frac{1}{x^2+8x+10}\,dx = \int \frac{1}{(x+4+\sqrt{6})(x-\sqrt{6}+4)} = \int \frac{A}{x-\sqrt{6}+4} + \frac{B}{x+\sqrt{6}+4}$, $D<0$

$x_{12} = \frac{-8 \pm \sqrt{64-40}}{2} = \frac{-8 \pm \sqrt{24}}{2} = \frac{-8 \pm 2\sqrt{6}}{2} = -4 \pm \sqrt{6}$

$\frac{A(x+\sqrt{6}+4) + B(x-6\cancel{*})}{(\ )(\ )} = \frac{(A+B)x - A(\sqrt{6}+\cancel{x}) + B(4-\sqrt{6})}{(\ )(\ )} =$

$A+B=0 \rightarrow B=-A$

$= A(\sqrt{6}+4) + B(4-\sqrt{6}) = 1$

$A(\sqrt{6}+4 - A(4+\sqrt{6}) = 1$

$2\sqrt{6}A = 1$

$A = \frac{1}{2\sqrt{6}}$

$B = -\frac{1}{2\sqrt{6}}$

$= \frac{1}{2\sqrt{6}}\int\left(\frac{1}{x-\sqrt{6}+4} - \frac{1}{x+\sqrt{6}+4}\right)dx =$

$\frac{1}{2\sqrt{6}}\left(\ln|x-\sqrt{6}+4|\right) - \ln|x+\sqrt{6}+4|\big) = C$

$D = 0$

$$\int \frac{1}{x^2+2x+1}\,dx = \int \frac{1}{(x+1)^2}\,dx = \int (x+1)'(x+1)^{-2}\,dx =$$

$$\frac{(x+1)^{-1}}{-1} + C = \frac{-1}{x+1} + C$$

$$\boxed{\int \frac{\text{konstans}}{\text{másodfokú } P_2(x)}\,dx}$$

1, $P_2(x)$ diszkr. < 0

teljes négyzet $\rightarrow \int \frac{f'}{1+f^2} = \text{arctg}\, f + C$

2, $P_2(x)$ diszkr. > 0

$P_2(x)$ gyöktényezős alak $\Rightarrow$ parciális törtre bontás $\rightarrow \int \frac{f'}{f}$

3, $P_2(x)$ diszkr. $= 0$

$P_2(x) \quad (P_1(x))^2 \rightarrow \int f' \cdot f^{-2}\,dx = \ldots$

Helyettesítéses integrálás. (módszer)

$$\int f(g(x))\,dx = \int f(t)\cdot\left(g^{-1}(t)\right)'\,dt$$

$t = g(x)$

Pl.: $\int \sin(2x)\,dx = \frac{1}{2}\int 2\sin\underset{\text{(hely. nélkül)}}{(2x)}\,dx = \frac{1}{2}\int (2x)'\sin(2x)\,dx = -\frac{1}{2}\cos(2x) + C$

$(2x)' = 2 \qquad \int f'(x)\sin f(x)\,dx = -\cos f(x) + C$

$\int \sin(2x)\,dx = \int \sin t \cdot \frac{1}{2}\,dx = \frac{1}{2}(-\cos t) + c = \frac{1}{2}(-\cos 2x) + c =$

$t = 2x$

$x = \frac{t}{2}$

$x' = \left(\frac{t}{2}\right)' = \frac{1}{2}$

$= -\frac{1}{2}\cos 2x + c$

$\int e^{\sqrt{x}}\,dx = \int e^t \cdot 2t\,dx = 2te^t - \int 2e^t\,dt = 2te^t - 2e^t + c =$

$t = \sqrt{x}$

$t^2 = x$

$x' = 2t$

$f'(t) = e^t \qquad f(t) = e^t$

$g(t) = 2t \qquad g'(t) = 2$

$= 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$

$\boxed{\text{Hf.:}}$

a, $\arcsin 2x$ ✓

b, $x^2 e^{-x}$ ✓

c, $\frac{1}{1+\sqrt{x+1}}$

d, $\frac{x^3}{\sqrt{x-1}}$

e, $\frac{2x+3}{x^2-5x+4}$

f, $\frac{2x+11}{x^2+6x+13}$